UNIVERSIDADE ESTADUAL PAULISTA “Júlio de Mesquita Filho”
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ANÁLISE DE NOVOS MODELOS MATEMÁTICOS PARA O PROBLEMA DE PLANEJAMENTO DA EXPANSÃO DE SISTEMAS DE TRANSMISSÃO

Ilha Solteira

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Tese apresentada ao Programa de Pós-Graduação em Engenharia Elétrica da Universidade Estadual Paulista "Júlio de Mesquita Filho" – UNESP, Campus de Ilha Solteira, para preenchimento dos pré-requisitos parciais para obtenção do título de Doutor em Engenharia Elétrica. Área de Conhecimento: Automação.

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering in the Electrical Engineering Faculty of Ilha Solteira UNESP, São Paulo, Brazil.

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RESUMO

O estado da arte em modelagem matemática do problema de planejamento da expansão de sistemas de transmissão (TEP) é analisado nesta tese de doutorado. É proposto um novo modelo linear disjuntivo para o problema TEP baseado no conceito de sistemas de numeração binária para transformar o modelo linear disjuntivo convencional em um problema com um número de variáveis binárias e contínuas muito menores, assim como do número de restrições relacionadas com essas variáveis. Também é usada a fase construtiva da metaheurística GRASP e restrições adicionais, encontradas da generalização do equilibrio de fluxo de potência em uma barra ou conjunto de barras para reduzir o espaço de busca. Os resultados mostram a importância da estratégia de redução do espaço de busca do problema TEP para resolver os modelos de transporte e linear disjuntivo. O problema TEP multiestágio é modelado como um problema de programação linear binária mista e resolvido usando um solver do tipo branch and bound comercial com tempos de processamento relativamente baixos. Outro tópico pesquisado foi a alocação de dispositivos FACTS tais como os capacitores fixos em série (FSCs) para aproveitar melhor a capacidade de transmissão das linhas e adiar ou reduzir o investimento em novas linhas de transmissão em um ambiente de planejamento multiestágio. Assim, pode ser esperado uma excelente relação custo/benefício da integração de dispositivos FSCs no planejamento multiestágio da expansão de sistemas de transmissão. Os resultados encontrados usando alguns sistemas testes mostram que a inclusão de FSCs no problema TEP é uma estratégia válida e efetiva em investimento para as empresas transmissoras e para os responsáveis da expansão nacional do sistema elétrico.

Palavras-chave: Planejamento multiestágio de sistemas de transmissão. Modelo disjuntivo reduzido. Sistema de numeração binária. GRASP. Compensação série fixa.

ABSTRACT

State-of-the-art models for transmission expansion planning problem are provided in this thesis. A new disjunctive model for the TEP problem based on the concept of binary numerical systems is proposed in order to transform the conventional disjunctive model to a problem with many fewer binary and continuous variables as well as connected constraints. The construction phase of a greedy randomized adaptive search procedure (GRASP-CP) together with fence constraints, obtained from power flow equilibrium, are employed in order to reduce search space. The studies demonstrate that the proposed search space reduction strategy, has an excellent performance in reducing the search space of the transportation model and reduced disjunctive model of TEP problem. The multistage TEP problem is modeled as a mixed binary linear programming problem and solved using a commercial branch and bound solver with relatively low computational time. Another topic studied in this thesis, is the allocation of FACTS devices in TEP problem. FACTS devices such as fixed series capacitors (FSCs) are considered in the multistage TEP problem to beter utilize the whole transfer capacity of the network and, consequently, to postpone or reduce the investment in new transmission lines. An excellent benefit-cost ratio can be expected from integration of FSC in multistage transmission expansion planning. The results obtained by using some real test systems indicate that the inclusion of FSCs in the TEP problem is a viable and cost-effective strategy for transmission utilities and national planning bureaus.

KEYWORDS: Multistage transmission expansion planning. Reduced disjunctive model (RDM). Binary numeral system. GRASP-CP. Fixed series compensation (FSC).

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LIST OF ABBREVIATIONS

AC	Alternating current
AMPL	A mathematical programming language
BNS	Binary numeral system
DC	Direct current
DM	Disjunctive model representation of the DC TEP problem
GRASP	Greedy randomized adaptive search procedure
GRASP-CP	Greedy randomized adaptive search procedure construction phase
HLM	Hybrid linear model
KCL	Kirchhoff current law
LP	Linear Programming
MILP	Mixed integer linear programming Problem
MTEP	Multistage transmission Expansion Planning
MINLP	Mixed integer nonlinear programming Problem
RDM	Reduced Disjunctive model representation of the DC TEP problem
RCL	Restricted candidate list
SI	Sensitivity index
STEP	Static Transmission Expansion Planning
TEP	Transmission expansion planning (TEP)
TP	Transportation model of TEP problem
SHPP	Shortest Path Problem
ACN	AC model of the TEP problem in Normal form
ACM	AC model of the TEP problem in Matrix form
DCNL	DC model of the TEP problem with power Losses
SDLC	Simplified DC model of the TEP problem with power Losses
DC	Nonlinear DC model for the TEP problem
MDC	Nonlinear DC model of the Multistage TEP problem
MDM	Disjunctive representation of the Multistage DC TEP problem
FSCTEP	Fixed series compensation allocation and the TEP problem model
FSCMTEP	Fixed series compensation allocation and the multistage TEP problem model

LIST OF SYMBOLS

The main symbols used in this thesis are listed below for quick reference. Other symbols are defined as needed throughout the text. The model constants are defined using capital Latin letters and the variables are defined by lower-case Latin letters.

A. Parameters

B_{ij}^{l}	Susceptance of a line in corridor *ij* (℧)
$B_{ij}^{l,sh}$	Shunt susceptance of a line in corridor *ij* (℧)
$B_{i}^{b,sh}$	Shunt susceptance of bus *i* (℧)
C_{ij}	Cost of a candidate line that can be added to corridor *ij* (US$)
C_{s}	The ratio of s^{th} FSC cost to candidate line cost (%)
E_{ij}^{c}	State of an existing line, candidate line or FSC in a corridor *ij* and condition *c*
G_{ij}^{l}	Series conductance of a line in corridor *ij* (℧)
$G_{i}^{b,sh}$	Shunt conductance of bus *i* (℧)
M	Large enough positive constant
N_{ij}^{0}	Number of existing lines in corridor *ij*
$\overline{N_{ij}}$	Maximum number of lines that can be added to corridor *ij*
ρ_{s}	Compensation level of s^{th} FSC in a candidate line (%).
P_{i}^{load}	Predicted power demand at bus *i (*MW*)*
$P_{i,t}^{load}$	Predicted active power demand at bus *i* and stage *t* (MW)
p_{i}^{g}	Predefined active power generation at bus *i* (MW)
$P_{i,t}^{g}$	Predefined active power generation at bus *i* and stage *t* (MW)
$\overline{P}_{i}^{g}$	Maximum active power generation at bus *i* (MW)
$\underline{P}_{i}^{g}$	Minimum active power generation at bus *i* (MW)
$\overline{P}_{ij}$	Maximum active power flow in corridor *ij* (MW)
$\overline{P}_{ij}^{c}$	Maximum active power flow in corridor *km* and condition *c* (MW).
Q_{i}^{d}	Predicted reactive power demand at bus *i* (MVAr)
$\overline{Q}_{i}^{g}$	Maximum reactive power generation at bus *i* (MVAr)
$\underline{Q}_{i}^{g}$	Minimum reactive power generation at bus *i* (MVAr)
$\overline{V}_{i}$	Maximum voltage magnitude at bus *i* (Volt)
$\underline{V}_{i}$	Minimum voltage magnitude at bus *i* (Volt)
$\overline{N}_{ij}$	Maximum number of the candidate lines in corridor *ij*

R_{ij} Resistance of a line in corridor *ij* (Ω)

X_{ij} Reactance of a line in corridor *ij* (Ω)

Y_{ij} Admittance of a line in corridor *ij* (℧)

Z_{ij} Impedance of a line in corridor *ij* (Ω)

η_{ij} Maximum number of candidate lines in corridor *ij* in the reduced disjunctive model

$\bar{\theta}$ The maximum voltage angle difference prevailing at the ends of each existing line or any built prospective line (Rad)

λ_t Discount factor used to calculate the net present value of the investment at stage *t* (%)

$\bar{\rho}_s$ Maximum compensation level permitted for a line (%)

B. variables

b_{ij} The *ij*th element of the susceptance matrix of a network, obtained as a function of the candidate lines (℧)

f_{ij}^{0} Lossless active power flow in existing lines of corridor *ij* (MW)

$f_{ij,t}^{0}$ Lossless active power flow in existing lines of corridor *ij* at stage *t* (MW)

$f_{ij,t,c}^{0}$ Lossless active power flow in existing lines of corridor *ij* at stage *t* and in contingency condition *c* (MW)

f_{ij} Lossless power flow in candidate lines in corridor *ij* (MW)

$f_{ij,t}$ Lossless power flow in candidate lines in corridor *ij* and stage *t* (MW)

$f_{ji,y}$ Lossless power flow in candidate line *y* in corridor *ij* (MW)

$f_{ij,y,t}$ Lossless power flow in candidate line *y* in corridor *ij* at stage *t* (MW)

$f_{ij,y,t,c}$ Lossless power flow in candidate line *y* in corridor *ij* at stage *t* and in contingency condition *c* (MW)

g_{ij} The *ij*th element of the conductance matrixes of a network obtained as a function of the candidate lines (℧)

i_{ij} The electric current in a line of corridor *ij* (A)

n_{ij} Number of lines added to corridor *ij*

$n_{ij,t}$ Number of lines added to corridor *ij* in stage *t*

p_i Net injected active power flow to bus *i* (MW)

p_i^{g} Active power generation at bus *i* (MW)

$p_{i,t}^{g}$ Net injected active power flow in bus *i* and stage *t* (MW)

p_{ij} Active power flow in corridor *ij* (MW)

p_{ij}^{loss} Active power losses in corridor *ij* (MW)

p_{ij}^{total}	Active power flow in corridor *ij* with transformer (MW)
q_i	Net injected reactive power flow to bus *i* (MVAr)
q_i^g	Reactive power generation at bus *i* (MVAr)
q_{ij}	Reactive power flow in corridor *ij* (MVAr)
q_{ij}^{total}	Reactive power flow in corridor *ij* with transformer (MVAr)
v_i	voltage level at buses *i* (volt)
$w_{ij,s}$	Binary variable representing s^{th} FSC that can be integrated into existing and candidate lines of corridor *ij*
$x_{ij,y}$	Binary variable used to model candidate line *y* in corridor *ij*
$x_{ij,y,t}$	Binary variable used to model candidate line *y* in corridor *ij* in stage *t*
$z_{ij,t,s}$	Binary variable representing s^{th} FSC that can be integrated into existing lines of corridor *ij* at stage *t*
$\psi_{ij,s}^{0}$	Auxiliary variables used for linearization of power flow in existing lines
$\psi_{ij,t,\mathrm{c},s}^{0}$	Auxiliary variables used for linearization of power flow in existing lines
$\psi_{ij,y,t,c,s}$	Auxiliary variables used for linearization of power flow in candidate lines
θ_i	Voltage angle at bus *i (Rad)*
$\theta_{i,t}$	Voltage angle for bus *i* in stage *t* (Rad)
$\theta_{i,t,c}$	Voltage angle for bus *i* in stage *t* and condition *c* (Rad)

Sets:

C	Set of all system conditions
C^0	Normal system condition
C^1	Contingencies in existing lines
C^2	Contingencies in candidate lines
C^3	Contingency in FSCs
S	Set of all candidate FSCs
T	Set stages
Y	Set of binary variables for modeling lines
β	Set of all buses
Ω	Set of corridors
Ω_i	Set of lines connected to bus *i*

CONTENTS

INTRODUCTION

The electricity market is increasingly becoming competitive and demanding both technically and economically. In order to meet the ever-increasing demand from consumers, electric power companies need new planning tools in order to economically supply their consumers and technically maintain the power system operating stably and reliably.

With the invention of high performance computers in recent decades and advances in optimization techniques, power system planning is changing from experimental designing to intelligent and cost effective design. Power system planning is implemented in three main stages: 1) load forecasting, 2) generation expansion planning, and 3) transmission expansion planning. Once transmission planning is completed, reactive power planning, stability and reliability criteria and other short term analysis are carried out in order to assure that system operation is viable under real power system conditions. This thesis focuses on transmission expansion planning in which the transmission components are optimally allocated to the system.

Transmission system components transfer the power from generators to load centers. They are very expensive and must be chosen by means of precise mathematical and technical calculations. To accomplish this, a mathematical model of the problem is first defined; then technical considerations are added to the model, and, finally, the problem is solved by an optimization process. Since it is not possible to consider all technical issues in a single model, numerous mathematical models have been proposed for this problem, with different objectives and approaches (LATORRE et al., 2003; LEE et al., 2006; ROMERO et al., 2002). The research performed on the transmission network expansion planning (TEP) problem can be divided into three types of activity: (1) mathematical modeling (ROMERO et al., 2002; ALGUACIL et al., 2003; HASHIMOTO et al., 2003; TAYLOR; HOVER, 2011), (2) optimization techniques (RAHMANI et al., 2012; ROMERO et al., 2002; ROMERO et al., 2007; SOUSA; ASADA, 2012; SUM-IM et al., 2009), and (3) inclusion of practical issues in the TEP model related (BUYGI et al., 2004; TORRE et al., 2008; KAZEROONI; MUTALE, 2010; RAHMANI; et al., 2013; SILVA et al., 2006).

The main objective of the transmission expansion planning (TEP) problem in an electric power system is to define where, how many, and when new transmission components (lines, transformers, capacitors, etc.) must be added to the system in order to meet the predicted power demand and to assure its operation is viable for a pre-defined planning horizon at minimum cost. The problem of power system transmission expansion planning (TEP) is a multistage (dynamic) problem, in which the planning horizon is divided into several stages and new transmission components are installed in each stage (ESCOBAR et al., 2004; SUM-IM et al., 2009; VINASCO et al., 2011). However, for the sake of simplicity, some planners consider it as a single-stage (static) programming problem in that planning is carried out for the predicted demand in the last period. In this thesis, multistage and static planning are studied.

The mathematical model of the TEP problem is a mixed integer, nonlinear, non-convex optimization problem, which is a very complex and computationally demanding problem (ESCOBAR et al., 2004; VERMA et al., 2010). This problem presents many local optimal solutions, and when system size increases, the number of local solutions grows exponentially. Therefore, researchers usually employ a variety of approaches to obtain high quality solutions for this problem. Some examples include: classical methods (BINATO et al., 2001b; RIDER et al., 2008), heuristic algorithms (ROMERO et al., 2005; ROMERO et al., 2007), metaheuristic strategies (BINATO et al., 2001a; GALLEGO et al., 1997; RAHMANI et al., 2010b; VERMA et al., 2010), relaxed mathematical models (ROMERO et al., 2002; TAYLOR; HOVER, 2011) and hybrid methods (BALIJEPALLI; KHAPARDE, 2011; CHUNG et al., 2003; RAHMANI et al., 2010a). A comprehensive review of these strategies is provided in (LATORRE et al., 2003; LEE et al., 2006).

The DC model (ROMERO et al., 2002) is the most commonly used network model in transmission expansion planning studies since it can model the TEP problem with acceptable precision and it can be solved much more easily than a complete AC model (JABR, 2013; RAHMANI et al., 2010a; RIDER et al., 2007; TAYLOR; HOVER, 2012). But the DC model is still a mixed integer nonlinear programming (MINLP) problem. There are numerous methods for solving this problem. However, with the impressive development in solving methodology, it is not possible to obtain the optimum solution for large or even medium-sized systems in polynomial time. Therefore, the investigation of new methodologies for solving this complicated problem is still a very attractive research area. Metaheuristic and classical algorithms are the most successful methods proposed to directly solve the nonlinear DC model. However, metaheuristics are not able to guarantee the optimality of a solution and classical methods such as branch and bounds and Benders decomposition can propose optimum solutions only when the model is convex.

An alternative to the DC model is the linear disjunctive model (DM). In this case, the planning problem is a mixed binary linear programming problem and has been verified that the optimum solution with this model is also optimum for the nonlinear DC model (PEREIRA; GRANVILLE, 1985; TSAMASPHYSROU et al., 1999; BAHIENSE et al., 2001; BINATO et al., 2001b). The DM has been employed for studying the static TEP in a number of studies. Meanwhile, multistage TEP using the DM is in an initial stage and little technical literature on the subject exists (VINASCO et al., 2011).

The extension to multistage increases the number of continuous and binary variables and network constraints. As a consequence, the planning problem rapidly becomes intractable by integer programming techniques. Even in a static TEP, when the size of the problem grows, access to the optimum solution problem becomes difficult. The size of the problem grows with power system size, the number of dispatch scenarios, and the number of probable contingencies scenarios. Therefore, depending on the size of the problem, the reduction of the search space of the problem may be needed.

1.1 OBJECTIVES

The main objectives of this study are:

a) To present the main models for the transmission expansion planning problem.

b) To propose new models for this problem.

c) To propose GRASP as a metaheuristic to reduce the search space of the problem.

d) To introduce valid inequalities (called *fence constraints* in this thesis) that help accelerate the convergence of the problem when the mixed integer programming problem is solved using branch and bound methods.

e) To include fixed series compensations in transmission lines to postpone/reduce investment costs in new transmission lines.

1.2 ORGANIZATION

In Chapter 2 basic models for transmission expansion problem as well as the latest models are presented. The mathematical formulation for AC model, both in its original and matrix form, the DC model, both in nonlinear and disjunctive form, the hybrid model and transportation model are given. Some models are presented for both the static and multistage problem.

In Chapter 3 some novel strategies to reduce the number of variables and the combinatorial search space in static and multistage transmission expansion planning problem are discussed. The concept of the binary numeral system is used to reduce the number of binary variables related to candidate transmission lines as well as continuous variables and network constraints. The GRASP construction phase and fence constraints, obtained from power flow equilibrium, are employed to reduce the combinatorial search space of the TEP problem.

In Chapter 4 a mathematical model for multistage transmission expansion planning (TEP) considering fixed series compensation (FSC) allocation and N-1 security constraints for both transmission lines and FSCs is proposed. FSCs are considered to increase transmission lines transfer capacity but the importance of using them in the TEP is to dispatch the power more efficiently, resulting in a different topology with less investment cost compared with planning without FSCs. This problem is modeled as a linear mixed binary programming problem and is solved by a commercial branch and bound solver to obtain the optimum solution. FSC is modeled as fixed impedance located in the transmission lines in discrete steps with an investment cost based on the prices of reinforcement lines.

In chapter 5 the proposed strategy for reducing the search space of the TEP problem is employed to solve the transportation, common disjunctive model (DM), and reduced disjunctive model (RDM) of the transmission expansion planning problem. In addition, two known test systems are analyzed to show the economic benefits of using fixed series compensations in multistage TEP problems.

STATE-OF-THE-ART MODELS FOR TRANSMISSION EXPANSION PLANNING

2.1 INTRODUCTION

The history of transmission expansion planning dates back to the decade of the 1970s when the first model was proposed (GARVER, 1970). Garver used a heuristic method to add transmission lines in an iterative process. In order to add a line to the system, the path with the highest overload level was identified by a linear programming (LP) problem. Since then, a great deal of effort and research has gone into solving this problem or improving the mathematical models.

Although the principal problem of transmission planning has a single definition, there are several models for this problem because it is not practical to adopt a single model and solve the problem in one step. This section presents the state-of-the-art models for the TEP problem. The investment cost of transmission lines is considered as an objective function and basic optimal power flow equations are considered as the constraints for the problem. In section 2.2, the complete set of models for static planning is provided: the AC model (both in its original and matrix form), the DC model with and without power losses, the hybrid model and the transportation model. For multistage planning, only the DC model of the problem is provided both in nonlinear and disjunctive form. The multistage version of the AC model is very difficult and there are no studies on this problem. The multistage model for hybrid and transportation models can be easily derived from the multistage DC model.

2.2 STATIC MODELS FOR TRANSMISSION EXPANSION PLANNING

2.2.1 AC Models for the TEP Problem

2.2.1.1 AC model for the TEP problem in normal form

The full static AC model of the transmission expansion planning problem in normal form (ACN) is provided in equations (1a)-(1o). These equations are obtained based on the basic equation of power flow presented in the Appendix A. The power flow in transmission lines and the power balances in system buses are in Appendix A, equations (34) to (39).

ACN: $Min\ v_1 = \sum_{ij \in \Omega} C_{ij} n_{ij}$ (1a)

s.t.

$$p_i^g - G_i^{b,sh} v_i^2 - \sum_{ij \in \Omega} p_{ij}^d + \sum_{ji \in \Omega} p_{ji}^r = P_i^{load} \qquad \forall i \in \beta \qquad (1b)$$

$$q_i^g - B_i^{b,sh} v_i^2 - \sum_{ij \in \Omega} q_{ij}^d + \sum_{ji \in \Omega} q_{ji}^r = Q_i^{load} \qquad \forall i \in \beta \qquad (1c)$$

$$p_{ij}^d = (n_{ij} + N_{ij}^0)[G_{ij}^l v_i^2 - v_i v_j (G_{ij}^l \cos\theta_{ij} + B_{ij}^l \sin\theta_{ij})] \qquad \forall ij \in \Omega \qquad (1d)$$

$$p_{ij}^r = (n_{ij} + N_{ij}^0)[G_{ij}^l v_j^2 - v_i v_j (G_{ij}^l \cos\theta_{ij} - B_{ij}^l \sin\theta_{ij})] \qquad \forall ij \in \Omega \qquad (1e)$$

$$q_{ij}^d = (n_{ij} + N_{ij}^0)[-(B_{ij}^l + \frac{B_{ij}^{l,sh}}{2}) v_i^2 - v_i v_j (G_{ij}^l \sin\theta_{ij} - B_{ij}^l \cos\theta_{ij})] \qquad \forall ij \in \Omega \qquad (1f)$$

$$q_{ij}^r = (n_{ij} + N_{ij}^0)[-(B_{ij}^l + \frac{B_{ij}^{l,sh}}{2}) v_j^2 + v_i v_j (G_{ij}^l \sin\theta_{ij} + B_{ij}^l \cos\theta_{ij})] \qquad \forall ij \in \Omega \qquad (1g)$$

$$\underline{P}_i^g \le p_i^g \le \overline{P}_i^g \qquad \forall i \in \beta \qquad (1h)$$

$$\underline{Q}_i^g \le q_i^g \le \overline{Q}_i^g \qquad \forall i \in \beta \qquad (1i)$$

$$\underline{V}_i \le v_i \le \overline{V}_i \qquad \forall i \in \beta \qquad (1j)$$

$$\sqrt{(p_{ij}^d)^2 + (q_{ij}^d)^2} \le (n_{ij} + N_{ij}^0)\overline{P_{ij}} \qquad \forall ij \in \Omega \qquad (1k)$$

$$\sqrt{(p_{ij}^r)^2 + (q_{ij}^r)^2} \le (n_{ij} + N_{ij}^0)\overline{P_{ij}} \qquad \forall ij \in \Omega \qquad (1l)$$

$$-\overline{\theta} \le \theta_{ij} \le \overline{\theta} \qquad if\ (N_{ij}^0 + n_{ij}) \ge 1 \qquad \forall ij \in \Omega \qquad (1m)$$

$$0 \le n_{ij} \le \overline{N_{ij}} \qquad \forall ij \in \Omega \qquad (1n)$$

$$n_{ij} \in \{0,1,2,...\} \qquad \forall ij \in \Omega \qquad (1o)$$

Equation (1a) is an objective function related to the investment costs of transmission lines. Equations (1b) and (1c) respectively represent active and reactive power balances. Equations

(1d) and (1e) expresses direct and reverse active power flows in transmission lines. The number of candidate transmission lines is given by n_{ij} while existing transmission lines are given by N_{ij}^{0}. Equations (1f) and (1g) expresses direct and reverse reactive power flows in transmission lines. The generation limits for active and reactive power sources are stated by (1h) and (1i), and for the voltage magnitudes by (1j). The limits (MVA) of the flows are represented by (1k) and (1l). The voltage angle difference between two buses with a line connecting them is limited by (1m) (CARPENTIER, 1979). According to Cain (2012), equation (1m) is needed for stability reasons and based on the theoretical steady-state stability limit, the angle difference between two buses with transmission lines is not greater than 90 degrees. The maximum transfer capacity of the transmission lines is stated by (1n) and finally the integer nature of the transmission lines is stated by (1o).

It is also possible to present the model with transformers. In this case, the active and reactive power flow in transmission lines are expressed in equations (2a)-(2d). The rest of constraints remain unchanged.

$$p_{ij}^{d,total} = (n_{ij} + N_{ij}^{0})[G_{ij}^{l}(\mathrm{A}_{ij}\, v_i)^2 - A_{ij} v_i v_j (G_{ij}^{l}\cos(\theta_{ij} + \varphi_{ij}) + B_{ij}^{l}\sin(\theta_{ij} + \varphi_{ij}))] \qquad \forall ij \in \Omega \qquad (2a)$$

$$p_{ij}^{r,total} = (n_{ij} + N_{ij}^{0})[G_{ij}^{l} v_j^2 - A_{ij} v_i v_j (G_{ij}^{l}\cos(\theta_{ij} + \varphi_{ij}) - G_{ij}^{l}\sin(\theta_{ij} + \varphi_{ij}))] \qquad \forall ij \in \Omega \qquad (2b)$$

$$q_{ij}^{d,total} = (n_{ij} + N_{ij}^{0})[-(B_{ij}^{l} + \frac{B_{ij}^{l,sh}}{2})(\mathrm{A}_{ij}\, v_i)^2 - A_{ij} v_i v_j (G_{ij}^{l}\sin(\theta_{ij} + \varphi_{ij}) - B_{ij}^{l}\cos(\theta_{ij} + \varphi_{ij}))] \; \forall ij \in \Omega \qquad (2c)$$

$$q_{ij}^{r,total} = (n_{ij} + N_{ij}^{0})[-v_j^2(B_{ij}^{l} + \frac{B_{ij}^{l,sh}}{2}) + A_{ij} v_i v_j (G_{ij}^{l}\sin(\theta_{ij} + \varphi_{ij}) + B_{ij}^{l}\cos(\theta_{ij} + \varphi_{ij}))] \qquad \forall ji \in \Omega \qquad (2d)$$

2.2.1.2 AC model in matrix form

It is possible to formulate the model in matrix form (ACM), where the power balances are expressed using the admittance matrix of the network. This model was first presented by (RIDER et al., 2007) and then used for further studies in (RAHMANI et al., 2010a; HOOSHMAND et al., 2012). Here, it is represent it in a more compact form.

$$\textbf{ACM}: \; Min\ v_1 = \sum_{ij \in \Omega} C_{ij} n_{ij} \qquad (3a)$$

s.t.

$$p_i^{g} - p_i = P_i^{load} \qquad \forall i \in \beta \qquad (3b)$$

$$q_i^g - q_i = Q_i^{load} \qquad \forall i \in \beta \qquad (3c)$$

$$p_i = v_i \sum_{j \in \beta} v_j [g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij}] \qquad \forall i \in \beta \qquad (3d)$$

$$q_i = v_i \sum_{j \in \beta} v_j [g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij}] \qquad \forall i \in \beta \qquad (3e)$$

$$g_{ij} = \begin{Bmatrix} -(n_{ij} + N_{ij}^0) G_{ij}^l) & \text{if } i \neq j \\ \sum_{j \in \Omega_i} (n_{ij} + N_{ij}^0) G_{ij}^l) & \text{if } i = j \end{Bmatrix} \qquad \forall ij \in \Omega \qquad (3f)$$

$$b_{ij} = \begin{Bmatrix} -(n_{ij} + N_{ij}^0) B_{ij}^l & \text{if } i \neq j \\ B_i^{b,sh} + \sum_{j \in \Omega_i} (n_{ij} + N_{ij}^0)(B_{ij}^l + \frac{B_{ij}^{l,sh}}{2}) & \text{if } i = j \end{Bmatrix} \qquad \forall ij \in \Omega \qquad (3g)$$

$$p_{ij}^d = (n_{ij} + N_{ij}^0)[G_{ij}^l v_i^2 - v_i v_j (G_{ij}^l \cos\theta_{ij} + B_{ij}^l \sin\theta_{ij})] \qquad \forall ij \in \Omega \qquad (3h)$$

$$p_{ij}^r = (n_{ij} + N_{ij}^0)[G_{ij}^l v_j^2 - v_i v_j (G_{ij}^l \cos\theta_{ij} - B_{ij}^l \sin\theta_{ij})] \qquad \forall ij \in \Omega \qquad (3i)$$

$$q_{ij}^d = (n_{ij} + N_{ij}^0)[-(B_{ij}^l + \frac{B_{ij}^{l,sh}}{2}) v_i^2 - v_i v_j (G_{ij}^l \sin\theta_{ij} - B_{ij}^l \cos\theta_{ij})] \qquad \forall ij \in \Omega \qquad (3j)$$

$$q_{ij}^r = (n_{ij} + N_{ij}^0)[-(B_{ij}^l + \frac{B_{ij}^{l,sh}}{2}) v_j^2 + v_i v_j (G_{ij}^l \sin\theta_{ij} + B_{ij}^l \cos\theta_{ij})] \qquad \forall ij \in \Omega \qquad (3k)$$

$$\underline{P}_i^g \le p_i^g \le \overline{P}_i^g \qquad \forall i \in \beta \qquad (3l)$$

$$\underline{Q}_i^g \le q_i^g \le \overline{Q}_i^g \qquad \forall i \in \beta \qquad (3m)$$

$$\underline{V_i} \le v_i \le \overline{V_i} \qquad \forall i \in \beta \qquad (3n)$$

$$\sqrt{(p_{ij}^d)^2 + (q_{ij}^d)^2} \le (n_{ij} + N_{ij}^0)\overline{P_{ij}} \qquad \forall ij \in \Omega \qquad (3o)$$

$$\sqrt{(p_{ij}^r)^2 + (q_{ij}^r)^2} \le (n_{ij} + N_{ij}^0)\overline{P_{ij}} \qquad \forall ij \in \Omega \qquad (3p)$$

$$-\overline{\theta} \le \theta_{ij} \le \overline{\theta} \qquad if\ (\ N_{ij}^0 + n_{ij}\ ) \ge 1 \qquad \forall ij \in \Omega \qquad (3q)$$

$$0 \le n_{ij} \le \overline{N_{ij}} \qquad \forall ij \in \Omega \qquad (3r)$$

$$n_{ij} \in \{0,1,2,...\} \qquad \forall ij \in \Omega \qquad (3s)$$

In this model p_i and q_i are the net injected active and reactive flow in bus i and used to represent the power balance equations (3b) and (3c). The expressions of p_i and q_i are given in (3d) and (3e) based on the admittance matrix of the network. The admittance matrix of the network is composed of two sub-matrices with elements b_{ij} and g_{ij} which respectively states the susceptance and conductance of the network. Equations (3f) and (3g) provide the expression of b_{ij} and g_{ij}, which are functions of the candidate transmission lines. Equations (3h)-(3s) were discussed in section 2.2.1.1.

2.2.2 DC Model with Power Losses

The AC model provided in the previous section is non-convex, nonlinear, and very complex. It is possible to simplify the AC model by adopting the following assumptions to obtain the DC model, with power losses.

1. The reactive power flow is dropped from the AC formulation. The transmission system is represented only considering the active part of the power while the reactive part is treated separately in later stages by reactive expansion planning.

2. Since the bus voltages of a system are near to 1.0 per-unit, the system voltages are fixed at this level. It is considered that in normal system conditions, the difference voltage angle difference prevailing at both ends of transmission lines is small. Therefore, the *sin(·)* function is considered to be equal to its argument, that is to say, $\sin\theta \cong \theta$.

3. Shunt conductance and susceptance of transmission lines are not considered in the model since they have little effect on transmission planning.

In order to provide a more simple model, the power flow in transmission lines is separated into two parts, lossless power flow and power losses. In this way, the lossless power flow becomes equal at the sending and receiving buses. The active power losses in a transmission line can be stated by summing the direct and reverse power flow (equations (1d) and (1e)) as follows.

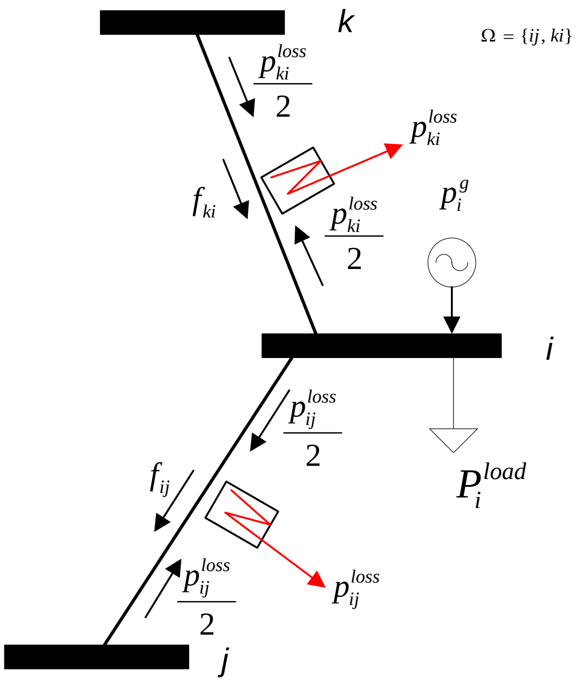
$$p_{ij}^{loss} = p_{ij}^{d} + p_{ji}^{r} = (n_{ij} + N_{ij}^{0})G_{ij}^{l}(v_i^2 + v_j^2 - 2v_i v_j \cos\theta_{ij}) \qquad \forall ij \in \Omega \qquad (4a)$$

Considering voltages at 1 p.u. we have:

$$p_{ij}^{loss} = 2(n_{ij} + N_{ij}^{0})G_{ij}^{l}(1 - \cos\theta_{ij}) \qquad \forall ij \in \Omega \qquad (4b)$$

Figure 1 shows the power balance in the DC TEP model using lossless power flow and power losses. The lossless power flow at the sending and receiving ends are equal, therefore, we only use a unique variable (f_{ij}) to model them. It is assumed that the power losses are procured by both ends of the transmission lines, each with a half contribution.

Figure 1 - The model and equations for power balance in the DC model with power losses



Source: The author

Therefore, the power balances in bus *i* can be expressed by equation (5).

$$p_i^g - f_{ij} + f_{ki} - \frac{1}{2}(p_{ij}^{loss} + p_{ki}^{loss}) = P_i^{load} \qquad \forall i \in \beta \tag{5}$$

Taking into account the discussion above, the Nonlinear DC model of the TEP problem with power losses (DCNL) is given in (6a)-(6j).

$$\textbf{DCNL:} \; Min \; v_1 = \sum_{ij \in \Omega} C_{ij} n_{ij} \tag{6a}$$

s.t.

$$p_i^g - \sum_{ij \in \Omega} f_{ij} + \sum_{ji \in \Omega} f_{ji} - \frac{1}{2}(\sum_{ij \in \Omega} p_{ij}^{ploss} + \sum_{ji \in \Omega} p_{ji}^{ploss}) = P_i^{load} \quad \forall i \in \beta \tag{6b}$$

$$f_{ij} = -B^{l}_{ij}(N^{0}_{ij} + n_{ij})\theta_{ij} \qquad \forall ij \in \Omega \qquad (6c)$$

$$p^{loss}_{ij} = 2G^{l}_{ij}(N_{ij}{}^{0} + n_{ij})[1 - \cos\theta_{ij}] \qquad \forall ij \in \Omega \qquad (6d)$$

$$f_{ij} + \frac{1}{2}p^{loss}_{ij} \le (N^{0}_{ij} + n_{ij})\overline{P_{ij}} \qquad \forall ij \in \Omega \qquad (6e)$$

$$-f_{ij} + \frac{1}{2}p^{loss}_{ij} \le (N^{0}_{ij} + n_{ij})\overline{P_{ij}} \qquad \forall ij \in \Omega \qquad (6f)$$

$$0 \le p^{g}_{i} \le \overline{P^{g}_{i}} \qquad \forall i \in \beta \qquad (6g)$$

$$-\overline{\theta} \le \theta_{ij} \le \overline{\theta} \qquad if\ (\ N^{0}_{ij} + n_{ij}\ ) \ge 1 \qquad \forall i \in \beta \qquad (6h)$$

$$0 \le n_{ij} \le \overline{N}_{ij} \qquad \forall ij \in \Omega \qquad (6i)$$

$$n_{ij} \in \{0,1,2,...\} \qquad \forall ij \in \Omega \qquad (6j)$$

where (6a) is investment in transmission lines. Constraint (6b) represents the power balance at each node. Constraints (6c) and (6d) demonstrate lossless power flow and active power losses in corridor *ij*, respectively. The power flow limit (for both power losses and lossless power flow) in corridor *ij* is given by (6e) and (6f). Equations (6g)-(6j) were already introduced in the previous subsections. It is possible to transfer this model to a mixed integer linear programming problem using the big-M technique (PEREIRA; GRANVILLE, 1985; TSAMASPHYSROU et al., 1999). A detailed discussion of linearizing this model can be found in (ALGUACIL et al., 2003; RAHMANI et al., 2013a).

2.2.3 DC Model

The DC model of transmission expansion planning (without power losses) is the most used model in transmission planning and numerous publications discuss this model. There are two different mathematical optimization models for the DC model: one is nonlinear and the other is an equivalent linear disjunctive model. These models are discussed in following sections.

2.2.3.1 DC model - nonlinear representation

Even with the many simplifications described in section 2.2.2, the DCNL model is a mixed integer nonlinear problem, nonconvex and highly complicated. The DCNL model can be simplified further by ignoring power losses. The nonlinear DC model for the TEP problem without power losses is obtained and presented in (6k)-(6r). In this model, all transmission lines obey two Kirchhoff's laws.

$$\textbf{DC: } Min\ v_1 = \sum_{ij \in \Omega_d} C_{ij} n_{ij} \tag{6k}$$

s.t.

$$p_i^g - \sum_{ij \in \Omega} f_{ij} + \sum_{ji \in \Omega} f_{ji} = P_i^{load} \qquad \forall i \in \beta \tag{6l}$$

$$f_{ij} = -B_{ij}^l (N_{ij}^0 + n_{ij}) \theta_{ij} \qquad \forall ij \in \Omega \tag{6m}$$

$$-(N_{ij}^0 + n_{ij}) \overline{P_{ij}} \le f_{ij} \le (N_{ij}^0 + n_{ij}) \overline{P_{ij}} \qquad \forall ij \in \Omega \tag{6n}$$

$$0 \le p_i^g \le \overline{P_i^g} \qquad \forall i \in \beta \tag{6o}$$

$$-\bar{\theta} \le \theta_{ij} \le \bar{\theta} \qquad if\ (\ N_{ij}^0 + n_{ij}\ ) \ge 1 \qquad \forall ij \in \Omega \tag{6p}$$

$$0 \le n_{ij} \le \overline{N}_{ij} \qquad \forall ij \in \Omega \tag{6q}$$

$$n_{ij} \in \{0,1,2,...\} \qquad \forall ij \in \Omega \tag{6r}$$

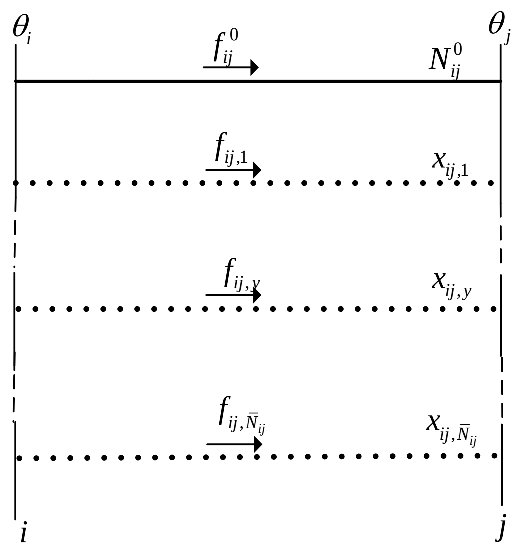
The nonlinearity of the DC model arises from the product of the voltage angle difference θ_{ij} and candidate transmission lines n_{ij} as stated in (6c).

2.2.3.2 DC model - disjunctive representation

The nonlinear DC model of transmission expansion planning can be transformed to a mixed integer linear model. This model is called a disjunctive model (DM) and separately proposed by Pereira (1985) and Tsamasphysrou (1999). It is always possible to transform a mixed integer nonlinear model with bilinear equations to a linear problem with binary variables using a large enough disjunctive coefficient (*M*). In order to provide the DM initially we present the transmission lines model between two buses as in Figure 2, where the existing transmission lines and their flows are N_{ij}^0, and f_{ij}^0, respectively. The binary variables for candidate transmission lines are $x_{ij,1},\ x_{ij,2},\ ...,\ x_{ij,y},....,x_{ij,\bar{N}_{ij}}$ where *y* is the *y*th candidate line in this corridor. The power flows in these transmission lines are expressed by continuous variables $f_{ij,1},\ f_{ij,2},\ ...,\ f_{ij,y},....,\ f_{ij,\bar{N}_{ij}}$.

In the DM, a binary variable is considered for each candidate line. This differs from the DC model where an integer variable was used to represent all the lines in a corridor. The number of binary variables for the lines in each candidate corridor is equal to the maximum number of transmission lines ($\overline{N}_{ij}$), allowed for installation, in that corridor. This results in a large number of binary variables. The disjunctive model of the TEP problem (DM) is provided in equations (7a)-(7l).

Figure 2 - Transmission lines model between two buses in the disjunctive model of TEP problem



Source: The author

DM: $Min\ v = \sum_{ij \in \Omega} C_{ij} \sum_{y \in Y} x_{ij,y}$ (7a)

$$p_i^g - \sum_{ij \in \Omega} \left(f_{ij}^0 + \sum_{y \in Y} f_{ij,y} \right) + \sum_{ji \in \Omega} \left(f_{ji}^0 + \sum_{y \in Y} f_{ji,y} \right) = P_i^{load} \qquad \forall i \in \beta \tag{7b}$$

$$f_{ij}^0 = -B_{ij}^l N_{ij}^0 \theta_{ij} \qquad \forall ij \in \Omega \tag{7c}$$

$$-N_{ij}^0 \overline{P}_{ij} \le f_{ij}^0 \le N_{ij}^0 \overline{P}_{ij} \qquad \forall ij \in \Omega \tag{7d}$$

$$-M\left(1 - x_{ij,y}\right) \le \frac{f_{ij,y}}{B_{ij}^l} + \theta_{ij} \le M\left(1 - x_{ij,y}\right) \qquad \forall ij \in \Omega, \forall y \in Y \tag{7e}$$

$$-\overline{P}_{ij} x_{ij,y} \le f_{ij,y} \le \overline{P}_{ij} x_{ij,y} \qquad \forall ij \in \Omega, \forall y \in Y \tag{7f}$$

$$x_{ij,y} \leq x_{ij,y-1} \qquad \forall ij \in \Omega, \forall y \in Y, y > 1 \tag{7g}$$

$$0 \leq p_i^g \leq \overline{P_i^g} \qquad \forall i \in \beta \tag{7h}$$

$$-\bar{\theta} \leq \theta_{ij} \leq \bar{\theta} \qquad \forall ij \in \Omega, N_{ij}^0 \geq 1 \tag{7i}$$

$$-\bar{\theta} - M\left(1 - x_{ij,y}\right) \quad \leq \theta_{ij} \leq \bar{\theta} + M\left(1 - x_{ij,y}\right) \qquad \forall ij \in \Omega, \forall y \in Y, N_{ij}^0 = 0 \tag{7j}$$

$$\sum_{y \in Y} x_{ij,y} \leq \bar{N}_{ij} \qquad \forall ij \in \Omega \tag{7k}$$

$$x_{ij,y} \; \{0,1\} \qquad \forall ij \in \Omega, \forall y \in Y \tag{7l}$$

In this model, (7a) stands for the investment in transmission lines. Constraint (7b) represents the power flow balance constraint or Kirchhoff's first law. Constraints (7c) and (7e), respectively, denote the expression of Ohm's law for the existing and candidate transmission lines in the equivalent DC network (Kirchhoff's second law) while constraints (7d) and (7f) are their power flow limit.

In (7e) and (7j) the value of the *M* should be large enough in order to maintain the feasible region of the problem. However, an *M* with a large value increases rounding errors and creates numerical instability problems, especially when the size of the system is large. The value of *M* can be selected better using the shortest path problem. The shortest path problem for the transmission expansion problem is presented in section A. IV. Using this method, the lowest value for *M* is generated without tightening the constraints and incurring aforementioned problems. Constraint (7g) guarantees sequential installation in the transmission line for each corridor. The maximum number of transmission lines is represented by constraint (7k).

2.2.4 Hybrid Linear Model

In the hybrid model, the power flows through circuits in the existing transmission lines are represented separately from the flows of candidate transmission lines. Therefore, the flow of existing transmission lines is represented by variable f_{ij}^0 and for candidate transmission lines, by f_{ij} . In the hybrid linear model only circuits of the base topology must follow the Kirchhoff's second law (6m). Ignoring this constraint for candidate transmission lines results in a linear model. With this assumption, the hybrid linear model (HLM) is provided in (8a)-(8h).

$$\textbf{HLM}\text{: } Min\ v_1 = \sum_{ij\in\Omega} C_{ij} n_{ij} \tag{8a}$$

s.t.

$$p_i^g - \sum_{ij\in\Omega}(f_{ij}^0 + f_{ij}) + \sum_{ji\in\Omega}(f_{ji}^0 + f_{ji}) = P_i^{load} \qquad \forall i \in \beta \tag{8b}$$

$$f_{ij}^0 = -B_{ij}^l N_{ij}^0 \theta_{ij} \qquad \forall ij \in \Omega \tag{8c}$$

$$-N_{ij}^0 \overline{P_{ij}} \le f_{ij}^0 \le N_{ij}^0 \overline{P_{ij}} \qquad \forall ij \in \Omega \tag{8d}$$

$$-n_{ij} \overline{P_{ij}} \le f_{ij} \le n_{ij} \overline{P_{ij}} \qquad \forall ij \in \Omega \tag{8e}$$

$$0 \le p_i^g \le \overline{P_i^g} \qquad \forall i \in \beta \tag{8f}$$

$$0 \le n_{ij} \le \overline{N}_{ij} \qquad \forall ij \in \Omega \tag{8g}$$

$$n_{ij} \in \{0,1,2,...\} \qquad \forall ij \in \Omega \tag{8h}$$

The advantage of the HLM over the DC model is that it is much easier to solve while the solution may be infeasible for the DC model. In the hybrid linear model, the constraint for the voltage angle difference is usually neglected.

2.2.5 Transportation Model

The transportation model was proposed by Garver (1970). For this model, Kirchhoff's second law, which is a nonlinear equation, is ignored. The transportation model of transmission expansion planning (TP) is given in (9a)-(9f).

$$\textbf{TP: } Min\ v_1 = \sum_{ij\in\Omega} C_{ij} n_{ij} \tag{9a}$$

s.t.

$$p_i^g - \sum_{ij\in\Omega} f_{ij} + \sum_{ji\in\Omega} f_{ji} = P_i^l \qquad \forall i \in \beta \tag{9b}$$

$$-(N_{ij}^0 + n_{ij})\overline{P_{ij}} \le f_{ij} \le (N_{ij}^0 + n_{ij})\overline{P_{ij}} \qquad \forall ij \in \Omega \tag{9c}$$

$$0 \le p_i^g \le \overline{P_i^g} \qquad \forall i \in \beta \tag{9d}$$

$$0 \le n_{ij} \le \overline{N}_{ij} \qquad \forall ij \in \Omega \tag{9e}$$

$$n_{ij} \in \{0,1,2,...\} \qquad \forall ij \in \Omega \tag{9f}$$

The great advantage of the transportation model is that it is mixed integer linear model since Kirchhoff's second law is omitted from the formulation both for existing and candidate transmission lines. The main disadvantage of the transportation model is that it is not able to find a feasible solution for the DC model.

2.3 MULTISTAGE MODEL FOR TRANSMISSION EXPANSION PLANNING

2.3.1 DC Model - Nonlinear Representation

The nonlinear representation of the DC model of multistage transmission expansion planning (MDC) (ESCOBAR et al., 2004) problem is shown in (10a)-(10h) where the related static model is obtained considering stage *T=1*.

$$\textbf{MDC:} \quad Min\ v = \sum_{t=1}^{T} \lambda_t \sum_{ij \in \Omega} C_{ij} n_{ij,t} \tag{10a}$$

s.t.

$$p_{i,t}^g - \sum_{ji \in \Omega} f_{ij,t} + \sum_{ij \in \Omega} f_{ji,t} = P_{i,t}^{load} \qquad \forall i \in \beta, \forall t \in T \tag{10b}$$

$$f_{ij,t} = -B_{ij}^l (N_{ij}^0 + \sum\nolimits_{k=1}^{t} n_{ij,k}) \theta_{ij,t} \qquad \forall ij \in \Omega, \forall t \in T \tag{10c}$$

$$-(N_{ij}^0 + \sum\nolimits_{k=1}^{t} n_{ij,k}) \overline{P_{ij}} \le f_{ij,t} \le (N_{ij}^0 + \sum\nolimits_{k=1}^{t} n_{ij,k}) \overline{P_{ij}} \qquad \forall ij \in \Omega, \forall t \in T \tag{10d}$$

$$0 \le p_{i,t}^g \le \overline{P_i^g} \qquad \forall i \in \beta, \forall t \in T \tag{10e}$$

$$-\overline{\theta} \le \theta_{ij,t} \le \overline{\theta} \qquad if\ (N_{ij}^0 + n_{ij,t}) \ge 1 \qquad \forall ij \in \Omega, \forall t \in T \tag{10f}$$

$$0 \le n_{ij,t} \le \overline{n}_{ij} \qquad \forall ij \in \Omega, \forall t \in T \tag{10g}$$

$$n_{ij,t} \ \text{integer} \qquad \forall ij \in \Omega, \forall t \in T \tag{10h}$$

where (10a) denotes investment in transmission lines projected to the base year; (10b) represents the power flow balance constraint; (10c) denotes the expression of Ohm's law for the equivalent DC network; (10d) and (10g) are the limits of the power flow of each corridor and transmission line.

2.3.2 DC Model - Disjunctive Representation

The model in (10a)-(10h) is a mixed integer nonlinear programming (MINLP) problem and highly complicated to solve. The multistage disjunctive model of TEP (MDM) is proposed in (VINASCO et al., 2011) to obtain the optimum solution of the problem.

$$\textbf{MDM:}\ \text{Min } v = \lambda_1 \sum_{ij\in\Omega} C_{ij} \sum_{y\in Y} x_{ij,y,1} + \sum_{t=2}^{T} \lambda_t \sum_{ij\in\Omega} C_{ij} \sum_{y\in Y} (x_{ij,y,t} - x_{ij,y,t-1}) \tag{11a}$$

$$p_{i,t}^{g} - \sum_{ij\in\Omega}\left(f_{ij,t}^{0} + \sum_{y\in Y} f_{ij,y,t} \right) + \sum_{ji\in\Omega}\left(f_{ji,t}^{0} + \sum_{y\in Y} f_{ji,y,t} \right) = P_{i,t}^{load} \quad \forall i \in \beta, \forall t \in T \tag{11b}$$

$$f^{0}{}_{ij,t} = -B_{ij} N_{ij}^{0} \theta_{ij,t} \qquad \forall ij \in \Omega, \forall t \in T \tag{11c}$$

$$-N_{ij}^{0} \overline{P_{ij}} \le f_{ij,t}^{0} \le N_{ij}^{0} \overline{P_{ij}} \qquad \forall ij \in \Omega, \forall t \in T \tag{11d}$$

$$-M\left(1 - x_{ij,y,t}\right) \le \frac{f_{ij,y,t}}{B_{ij}^{l}} + \theta_{ij,t} \le M\left(1 - x_{ij,y,t}\right) \quad \forall ij \in \Omega, \forall y \in Y, \forall t \in T \tag{11e}$$

$$-\bar{P}_{ij} x_{ij,y,t} \le f_{ij,y,t} \le P_{ij} x_{ij,y,t} \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T \tag{11f}$$

$$x_{ij,y,t} \le x_{ij,y-1,t} \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T \,/\, y > 1 \tag{11g}$$

$$x_{ij,y,t-1} \le x_{ij,y,t} \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T \,/\, t > 1 \tag{11h}$$

$$\sum_{y\in Y} x_{ij,y,t} \le \bar{N}_{ij} \qquad \forall ij \in \Omega, \forall t \in T \tag{11i}$$

$$0 \le p_{i,t}^{g} \le \overline{P_{i}^{g}} \qquad \forall i \in \beta, \forall t \in T \tag{11j}$$

$$-\bar{\theta} \le \theta_{ij,t} \le \bar{\theta} \qquad \forall ij \in \Omega, \forall t \in T, N_{ij}^{0} \ge 1 \tag{11k}$$

$$-\bar{\theta} - M\left(1 - x_{ij,y,t}\right) \le \theta_{ij,t} \le \bar{\theta} + M\left(1 - x_{ij,y,t}\right) \qquad \forall ij \in \Omega, \forall t \in T, \forall y \in Y, N_{ij}^{0} = 0 \tag{11l}$$

$$x_{ij,y,t} \text{ binary} \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T \tag{11m}$$

In this model, the objective function calculates the investment in transmission lines projected to the base year. The first term of the objective function calculates the cost of transmission lines for the first stage while the second term will be explained later when describing the constraint (11h). Constraint (11e) represents the power flow balance constraint or Kirchhoff’s first law. Constraints (11c) and (11e), respectively, denote the expression of Ohm’s law for the existing and candidate transmission lines while constraints (11d) and (11f) are their power flow limit.

Constraint (11g) avoids the same optimum solutions and it is very useful for converging the branch and bound method. Constraint (11h) guarantees that a line installed in a stage must be present in later stages. Therefore, the number of new transmission lines in stage t in corridor ij is calculated by $\sum_{y \in Y}(x_{ij,y,t} - x_{ij,y,t-1})$. The second part of the objective function uses this expression to calculate the cost of transmission lines for stage t. The maximum number of transmission lines is represented by constraint (11i). Constraints (11k) and (11l) are the maximum phase angle allowed in each bus and represents a stability constraint.

2.4 PRACTICAL CONSIDERATIONS IN TRANSMISSION EXPANSION PLANNING

There are many factors that may affect the best solution of transmission expansion planning. The dispatch scenarios, N-1/N-2 contingencies (SILVA et al., 2005), the uncertainty of future scenarios (MAGHOULI et al., 2011), the environmental impacts (KAZEROONI; MUTALE, 2010) and market considerations (TORRE et al., 2008) can change planning constraints or objectives. In some practical issues, such as N-1/N-2 contingency or uncertainty in generation or demand, the number of continuous variables and related constraints increases linearly depending on the occurrence of lines outages or a number of scenarios, respectively. In these cases, solving the problems becomes very challenging since it requires enormous memory size in order to save the branch and bound subproblems. Therefore, practical issues make the problem more complicated both in modeling and solving approaches.

Deregulation has also changed the structure of power systems, incorporating market issues into operation, planning and management. Due to open access and bid-based dispatch, generation and load patterns are likely to change more frequently and significantly (BUYGI et al., 2006; FANG; HILL, 2003; FOROUD et al., 2010; LATORRE et al., 2003; LEE et al., 2006). Deregulation has introduced new uncertainties for market participants and made the planning of transmission expansion more difficult. In addition, generation patterns can also be uncertain because of wind and photovoltaic power plants. The demand profile is also uncertain as a result of new loads such as plug-in electric vehicles, district heating, etc. Therefore, better transmission expansion planning methods are needed. There are currently a number of common techniques that can be applied in centralized and vertically integrated power systems (BAHIENSE et al., 2001; LEITE DA SILVA et al., 2011; RAHMANI et al., 2010b; ROMERO et al., 2003; ROMERO et al., 2007; SEIFI et al., 2007; SUM-IM et al., 2009; TAYLOR; HOVER, 2011; VERMA et al., 2010), but these methods might not be suitable for a competitive electricity market environment involving many possible future scenarios. A Scenario is defined as the potential of both future generation and demand. Any scenario might consider possible non-random uncertainties and thus involve potential installation/closures of generators as well as augmentation/reduction of the load demand. Several non-deterministic stochastic methods have been proposed to deal with TEP with multiple future scenarios (BUYGI et al., 2002, 2004; FANG; HILL, 2003; MAGHOULI et al., 2011; ZHAO et al., 2009, 2011). In non-deterministic approaches, the expansion plan is designed for all possible future scenarios where each scenario may have a different probability of occurrence. Scenario analysis and decision analysis are widely used to address such problems and are frequently applied to handle non-random uncertainties and consequently reduce the planning risk (FANG; HILL, 2003). Several probabilistic approaches have been proposed to deal with random uncertainties such as the uncertainties of load or generators (BUYGI et al., 2004). Stochastic programming can be used to find a feasible policy for most of the possible data instances and maximize the expectation of some functions that include both decisions and random variables (JIRUTITIJAROEN; SINGH, 2008; SINGH et al., 2010). A recent non-deterministic approach for transmission expansion planning with multiple scenarios was reported in (ZHAO et al., 2009, 2011) in which the most flexible expansion plan is considered as the plan that has the lowest adaptation cost. The adaption cost is an extra investment cost that should be reserved for transmission lines to cope with alternative scenarios.

STRATEGIES TO REDUCE THE NUMBER OF VARIABLES AND THE COMBINATORIAL SEARCH SPACE OF THE MULTISTAGE TRANSMISSION EXPANSION PLANNING PROBLEM

3.1 INTRODUCTION

In this chapter, some new strategies are proposed to reduce the number of variables and the combinatorial search space of the static and multistage transmission expansion planning problems. The concept of the binary numeral system (BNS) is used to reduce the number of binary, continuous variables and constraints related to the candidate transmission. The construction phase of the greedy randomized adaptive search procedure (GRASP-CP) and fence constraints, obtained from power flow equilibrium are employed to further reduce the search space. The static and multistage TEP problems are modeled as mixed binary linear programming problems and solved using a commercial solver.

Three approaches are proposed to reduce the search space of the TEP problem. The first and second approaches do not exclude the optimum solution from the feasible region; however, there is no guarantee that it will be possible to maintain the optimum solution in the feasible region using the third method. The methods are as follows:

1) A novel mathematical model is proposed in which the number of binary variables related to the candidate lines is reduced dramatically in the disjunctive representation of the DC TEP problem (DM). The concept of BNS is applied to transmission lines to decrease the number of candidate lines from $\bar{N}_{ij}$ in the DM to $\lceil \log_2(\bar{N}_{ij}+1) \rceil$ in the reduced disjunctive model (RDM). Consequently, the related continuous variables and constraints are reduced by this factor.

2) Some constraints obtained from the power flow balances are added to the model in order to enforce some of the decision variables to assume integer values when the relaxed LP is solved in the branch and bound algorithm. Although these constraints are redundant with respect to the MILP problem, they are important for the corresponding LP problem since they serve to reduce the gap between the optimal integer solution of the MILP problem and the solution

obtained by relaxing the integrality constraints (LP problem). In other words, these constraints can reduce the search space of the problem without affecting the feasible region of the MILP problem (HAFFNER et al., 2001; SOUSA; ASADA, 2012).

3) The GRASP-CP (BINATO et al., 2001a; FARIA et al., 2005; FEO; RESENDE, 1995) is used to identify the maximum limit of lines in each corridor in order to reduce the search space of the problem. The best solution over all the GRASP-CP iterations is considered to be the incumbent solution of the branch and bound algorithm and the maximum number of lines over all GRASP-CP iterations in each corridor is considered as the candidate line limit in that corridor. As a result, the search space is reduced significantly. This step may exclude the optimum solution from the search space of the problem if a small number of iterations are carried out and/or if the GRASP-CP is more greedy than random.

Once the search space has been reduced by these methods, a branch and bound algorithm (IBM ILOG CPLEX, 2012) can be used to solve the problem.

In this chapter, section 3.2 proposes a novel disjunctive mathematical model for the TEP problem using the concept of the binary numeral system. Section 3.3 introduces some fence constraints obtained from the power balances in system buses. The domain reduction using GRASP-CP is discussed in section 3.5.

3.2 TRANSMISSION EXPANSION PLANNING MODEL WITH REDUCED VARIABLES AND CONSTRAINTS

In the nonlinear DC model of the TEP problem (DC), the candidate transmission lines are modeled as integer variables in a decimal numeral system. It seems that the nonlinearity of the DC model cannot be removed without transferring it to a disjunctive model (DM), where for each candidate transmission line, a binary variable is used (BAHIENSE et al., 2001; BINATO et al., 2001b, PEREIRA; GRANVILLE, 1985). Although we have the advantage of avoiding the nonlinear model to a mixed integer linear one, the problem size also increased linearly with respect to the maximum number of transmission lines. In order to avoid dimensionality problems while maintaining the optimum solution, the disjunctive model is represented using the concept of the binary numeral system.

In the BNS, each integer variable is represented through a combination of binary variables. Therefore, the following binary expression is used to represent an integer variable

$$n_{ij} = \sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y} \quad x_{ij,y} \in \{0,1\} \tag{11}$$

where the number of candidate lines in corridor *ij* is an integer variable and $x_{ij,y}$ is a binary variable. With this method, every possible combination of candidate lines, with the same parameters (including investment cost, impedance, maximum flow capacity, etc) can be constructed with a reduced number of binary variables.

Example 1: Consider that there are a maximum number of 7 candidate transmission lines in a single corridor, i.e $n_{ij} = \{0,1,2,...,7\}$, all lines with \$50, 1℧ and 100 MW. As shown in Figure 3 in the DM, 7 binary variables represent these lines, while using the binary numeral system only three different binary variables are used in RDM as stated in (12) and indicated in Figure 3. In Figure 3 the first line has the same characteristics as the DM while the second and third lines of RDM have twice and quadruple the cost, capacity and susceptance of the first line.

$$n_{ij} = x_{ij,1} + 2x_{ij,2} + 4x_{ij,3} \qquad x_{ij,}, x_{ij,2}, x_{ij,3} \in \{0,1\} \tag{12}$$

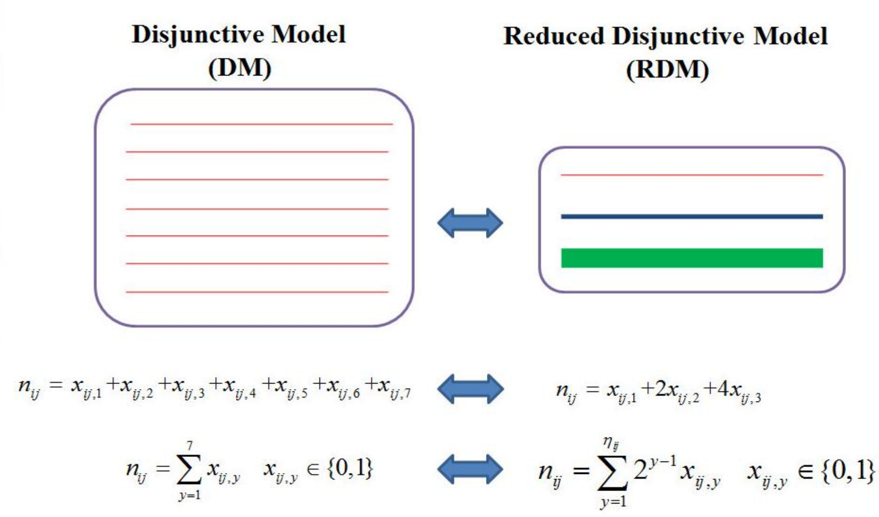
Table 1 shows the cost, the susceptance and the maximum flow capacity of binary variables (representing the transmission lines), obtained by multiplying the base line parameters by the coefficient provided in column 2. In the third rightmost columns, examples demonstrate how 2, 5, 7 lines are installed in a corridor.

Table 1 - Candidate lines modeling in the reduced disjunctive model

Binary variable	Coefficient	C_{ij}^{DCRDM} \$	B_{ij}^{DCRDM} ℧	$\bar{P}_{ij}^{DCRDM}$ MW	Examples of lines installed in a candidate path		
					2	5	7
1	2^0	50	1	100	1	1	0
2	2^1	100	2	200	1	0	1
3	2^2	200	4	400	0	0	0

Source: The author

Figure 3 - Comparison between candidate lines in the disjunctive model and the reduced disjunctive model



Source: The author

Transforming the candidate lines by equation (11) to binary variables becomes useful when the number of integer variables is greater than 2. Otherwise, the disjunctive model and the reduced disjunctive model become equal. As a general rule, with the BNS concept, a maximum number of *(2^n-1)* lines can be implemented with *n* variables, or $\eta_{ij} = \left\lceil \log_2(\bar{N}_{ij} + 1) \right\rceil$ binary variables are needed to implement a maximum number of $\bar{N}_{ij}$.

It is also important to mention that in addition to reducing the binary variables related to candidate transmission lines, the continuous variables and their network constraints are reduced by the same factor. The multistage of reduced dynamic disjunctive model (MRDM) with reduced binary and continuous variables as well as constraints using the concept of BNS is proposed in (13a)-(13l).

MRDM: $$\text{Min}\, v = \lambda_t \sum_{ij\in\Omega} C_{ij} \sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,1,y} + \sum_{t=2}^{T} \lambda_t \sum_{ij\in\Omega} C_{ij} \left(\sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t} - \sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t-1} \right) \quad (13a)$$

s.t.

$$p_{i,t}^{g} - \sum_{ij\in\Omega} \left(f_{ij}^{0} + \sum_{y=1}^{\eta_{ij}} f_{ij,y,t} \right) + \sum_{ji\in\Omega} \left(f_{ji,t}^{0} + \sum_{y=1}^{\eta_{ij}} f_{ji,y,t} \right) = P_{i,t}^{load} \quad \forall i \in \beta, \forall t \in T \quad (13b)$$

$$f_{ij,t}^{0} = -B_{ij}^{l} N_{ij}^{0} \theta_{ij,t} \qquad \forall ij \in \Omega, \forall t \in T \qquad (13c)$$

$$-N_{ij}^{0} \overline{P}_{ij} \leq f_{ij,t}^{0} \leq N_{ij}^{0} \overline{P}_{ij} \qquad \forall ij \in \Omega, \forall t \in T \qquad (13d)$$

$$-M\left(1 - x_{ij,y,t}\right) \leq \frac{f_{ij,y,t}}{2^{y-1} B_{ij}^{l}} + \theta_{ij,t} \leq M\left(1 - x_{ij,y}\right) \qquad \forall ij \in \Omega, \forall y \in \{1,...,\eta_{ij}\}, \forall t \in T \qquad (13e)$$

$$-2^{y-1} \overline{P}_{ij} x_{ij.y,t} \leq f_{ij,y,t} \leq 2^{y-1} \overline{P}_{ij} x_{ij,y,t} \qquad \forall ij \in \Omega, \forall y \in \{1,...,\eta_{ij}\}, \forall t \in T \qquad (13f)$$

$$\sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t-1} \leq \sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t} \qquad \forall ij \in \Omega, \forall t \in T \,/\, t > 1 \qquad (13g)$$

$$\sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t} \leq \overline{N}_{ij} \qquad \forall ij \in \Omega, \forall t \in T \qquad (13h)$$

$$0 \leq p_{i,t}^{g} \leq \overline{P_{i}^{g}} \qquad \forall i \in \beta, \forall t \in T \qquad (13i)$$

$$-\overline{\theta} \leq \theta_{ij,t} \leq \overline{\theta} \qquad \forall ij \in \Omega, \forall t \in T, N_{ij}^{0} \geq 1 \qquad (13j)$$

$$-\overline{\theta} - M\left(1 - x_{ij,y,t}\right) \leq \theta_{ij,t} \leq \overline{\theta} + M\left(1 - x_{ij,y,t}\right) \quad \forall ij \in \Omega, \forall t \in T, \forall y \in \{1,...,\eta_{ij}\}, N_{ij}^{0} = 0 \quad (13k)$$

$$x_{ij,y,t} \text{ binary} \qquad \forall ij \in \Omega, \forall y \in \{1,...,\eta_{ij}\}, \forall t \in T \qquad (13l)$$

In this model, the term 2^{y-1} has a key role in reducing the number of candidate binary variables ($x_{ij,y,t}$) and related continuous variables ($f_{ij,y,t}$) as well as constraints related to these variables (13e)-(13g). Constraint (13g) states that the sum of transmission lines presented in a stage must appear in a later stage. This is slightly different from the equivalent constraint in disjunctive model (7g), which enforces the same line installation of a stage to a later stage, and affects the objective function accordingly (compare the objective functions of MDM with MRDM). All the other constraints and equations of this model have a similar description to the MDM explained in section 2.3.2.

It should be noted N-1 security is one of the most important issues in transmission planning and has been addressed by a few papers (OLIVEIRA et al., 2004; SILVA et al., 2005; ZHANG et al., 2012). The N-1 contingency is not implemented in RDM model and will be treated in the future. In N-1 contingency, the number of continuous variables and related constraints increases linearly depending on the occurrence of lines outages. The RDM model is very promising for reduction of search space since it can decrease the number of continuous variables and their constraints, resulting in an efficient formulation while maintaining the optimum solution of the problem.

3.3 FENCE CONSTRAINTS

Adding some constraints obtained from Kirchhoff's current law (KCL) in a node and in general from the cutset in a supernode can improve the convergence rate of branch and bound significantly. These types of constraints were first used by Haffner (2001) and named as *fence constraints*. Although fence constraints are redundant with respect to the MILP, they are very important for the corresponding LP problem and its successors since it forces some of the decision variables to assume integer values without requiring that the corresponding separation be carried out (HAFFNER et al., 2001; SOUSA; ASADA, 2012). In other words, fence constraints can reduce the complexity of the problem. The best results will be obtained if these constraints are applied through branch and bound progress when a fractional part is observed in integer variables; however, for the sake of simplicity, these types of constraints are considered as fixed constraints from the beginning of problem. It should be noted that, in obtaining these constraints, we assumed that the generators are already defined by the power system market, therefore, the level of generators are considered to be fixed and stated by parameter $P_{i,t}^{g}$.

Below, in subsections 3.3.1 and 3.3.2, two types of constraints are added to the problem based on two different definitions of KCL in each node. Also, a generalization is made to apply these types of constraints to some supernodes in subsections 3.3.3 and 3.3.5. Supernodes can be created by combining neighboring nodes. In subsection 3.3.4, the efficiency of fence constraints is discussed.

3.3.1 Constraint Type-1 in a Node

The first set of constraints is based on the fact that the deficit of transmission capacity in a node must be provided by using new transmission lines connected to it considering the most favorable case (HAFFNER et al., 2001) and can be expressed by the following constraint:

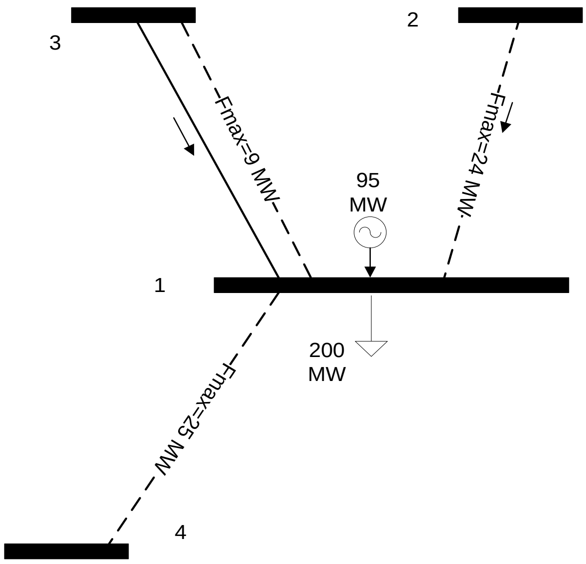
$$\sum_{ij\in\Omega}\sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t} \geq \left\lceil \frac{\left(\left| P_{i,t}^{g} - P_{i,t}^{load} \right| - \sum_{ij\in\Omega} N_{ij}^{0} \overline{P}_{ij} \right)}{\max\limits_{ij\in\Omega}(\overline{P}_{ij})} \right\rceil \quad \forall i \in \beta, \forall t \in T \tag{14a}$$

where a shortage of power in each node is calculated in the numerator of the right-hand side of the fraction and the most favorable case is obtained when dividing it by the maximum capacity of candidate lines connected to that node. Since the left-hand side is an integer, a ceiling is imposed on the right-hand side to adjust it to an integer value.

Example 2: In Figure 4 a generic system bus with the neighboring buses and candidate and existing transmission lines is illustrated. Consider the maximum number of candidate lines in this example to be 3 and the problem to be static (one stage). Therefore, using the model, two binary variables are sufficient for modeling these 3 lines. According to (14a), the following constraint is given in the bus number 1.

$$x_{12,1} + 2x_{12,2} + x_{13,1} + 2x_{13,2} + x_{14,1} + 2x_{14,2} \geq \left\lceil \frac{200-95-9}{25} \right\rceil = \lceil 3.84 \rceil = 4 \tag{14b}$$

Figure 4 - A generic system bus with neighboring buses and lines



Source: The author

3.3.2 Constraint Type-2 in a Node

Another perception of the KCL in each node can be stated as follows: the quantity of the lines connected to a certain bus must provide a transmission capacity larger or equal to the forecasted net injection in that bus (SOUSA; ASADA, 2012).

$$\sum_{ij\in\Omega}(N_{ij}^{0}+\sum_{y=1}^{\eta_{ij}}2^{y-1}x_{ij,y,t})\overline{P}_{ij}\geq P_{i,t}^{load}-P_{i,t}^{g}\qquad\forall\ i\in\beta,\forall t\in T \tag{15a}$$

However, this constraint is not very strong since there are fractional parts in both sides. Therefore, to create a stronger constraint, the following simplifications and approximations are provided. First, the constants are moved to the right side:

$$\sum_{ij\in\Omega}(\sum_{y=1}^{\eta_{ij}}2^{y-1}x_{ij,y}^{t})\overline{P}_{ij}\geq P_{i,t}^{load}-P_{i,t}^{g}-\sum_{ij\in\Omega}N_{ij}^{0}\overline{P}_{ij}\qquad\forall i\in\beta,\forall t\in T \tag{15b}$$

then considering $\Gamma=V(i)-\{ij\}$, $V(i)$ representing the lines connected to bus i, for each $ij\in V(i)$ and $\forall \mathrm{t}\in\mathrm{T}$, we have:

$$\sum_{y=1}^{\eta_{ij}}2^{y-1}x_{ij,y,t}\overline{P}_{ij}+\sum_{km\in\Gamma}\sum_{y=1}^{\eta_{km}}2^{y-1}x_{km,y,t}\overline{P}_{km}\geq P_{i,t}^{load}-P_{i,t}^{g}-\sum_{km\in\Gamma}N_{km}^{0}\overline{P}_{km}-N_{ij}^{0}\overline{P}_{ij} \tag{15c}$$

Dividing this equation by $\overline{P}_{ij}$; therefore, for each $ij\in V(i)$ and $\forall \mathrm{t}\in\mathrm{T}$ equation (15c) comes to the following:

$$\sum_{y=1}^{\eta_{ij}}2^{y-1}x_{ij,y,t}+\frac{\sum_{km\in\Gamma}\sum_{y=1}^{\eta_{km}}2^{y-1}x_{km,y,t}\overline{P}_{km}}{\overline{P}_{ij}}\geq\frac{P_{i,t}^{load}-P_{i,t}^{g}-\sum_{km\in\Gamma}N_{km}^{0}\overline{P}_{km}}{\overline{P}_{ij}}-N_{ij}^{0} \tag{15d}$$

and since $\sum_{km\in\Gamma}\sum_{y=1}^{\eta_{km}}2^{y-1}x_{km,y,t}\left\lceil\frac{\overline{P}_{km}}{\overline{P}_{ij}}\right\rceil\geq\frac{\sum_{km\in\Gamma}\sum_{y=1}^{\eta_{km}}2^{y-1}x_{km,y,t}\overline{P}_{km}}{\overline{P}_{ij}}$, constraint (15d) can be rewritten for $ij\in V(i)$ and $\forall \mathrm{t}\in\mathrm{T}$ as (15e).

$$\sum_{y=1}^{\eta_{ij}}2^{y-1}x_{ij,y,t}+\sum_{km\in\Gamma}\sum_{y=1}^{\eta_{km}}2^{y-1}x_{km,y,t}\left\lceil\frac{\overline{P}_{km}}{\overline{P}_{ij}}\right\rceil\geq\frac{P_{i,t}^{load}-P_{i,t}^{g}-\sum_{km\in\Gamma}N_{km}^{0}\overline{P}_{km}}{\overline{P}_{ij}}-N_{ij}^{0} \tag{15e}$$

The left-hand side of the above equation is an integer and since all the coefficients as well as the variables are integers, the right-hand side is also an integer. Finally for $\forall t \in T$ and $\forall ij \in V(i)$, the following constraint can be added to the model:

$$\sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t} + \sum_{km\in\Gamma} \sum_{y=1}^{\eta_{km}} 2^{y-1} x_{km,y,t} \left\lceil \frac{\overline{P}_{km}}{\overline{P}_{ij}} \right\rceil \geq \left\lceil \frac{P_{i,t}^{load} - P_{i,t}^{g} - \sum_{km\in\Gamma} N_{km}^{0} \overline{P}_{km}}{\overline{P}_{ij}} \right\rceil - N_{ij}^{0} \tag{15f}$$

When the constraint is added for the candidate line with largest maximum power flow constraints (15f) and (14a) become identical, in another words, constraint Type-1 is a special case of constraint Type-2.

Example 3: Considering Figure 4 and the assumption made in Example 2, the following constraints can be added to the problem for each neighboring line connected to bus number 1. There are 3 corridors connected to bus 1. Therefore, 3 constraints can be given according to equation (15f).

$$(x_{12,1} + 2x_{12,2}) + \left\lceil \frac{9}{24} \right\rceil (x_{13,1} + 2x_{13,2}) + \left\lceil \frac{25}{24} \right\rceil (x_{14,1} + 2x_{14,2}) \geq \left\lceil \frac{200-95-9}{24} \right\rceil$$

$$\Rightarrow \quad (x_{12,1} + 2x_{12,2}) + (x_{13,1} + 2x_{13,2}) + (2x_{14,1} + 4x_{14,2}) \geq 4 \tag{15g}$$

$$\left\lceil \frac{24}{9} \right\rceil (x_{12,1} + 2x_{12,2}) + (x_{13,1} + 2x_{13,2}) + \left\lceil \frac{25}{9} \right\rceil (x_{14,1} + 2x_{14,2}) \geq \left\lceil \frac{200-95}{9} \right\rceil - 1$$

$$\Rightarrow \quad (3x_{12} + 6x_{12,2}) + (x_{13,1} + 2x_{13,2}) + (3x_{14,1} + 6x_{14,2}) \geq 13 \tag{15h}$$

$$\left\lceil \frac{24}{25} \right\rceil (x_{12,1} + 2x_{12,2}) + \left\lceil \frac{9}{25} \right\rceil (x_{13,1} + 2x_{13,2}) + (x_{14,1} + 2x_{14,2}) \geq \left\lceil \frac{200-95-9}{25} \right\rceil$$

$$\Rightarrow \quad (x_{12,1} + 2x_{12,2}) + (x_{13,1} + 2x_{13,2}) + (x_{14,1} + 2x_{14,2}) \geq 4 \tag{15i}$$

3.3.3 Constraint Type-1 in a Supernode

A power system can be represented by a graph, $G = (\beta, \Omega)$, where β is the set of nodes and Ω is the set of corridors. Suppose that $P = (\beta_p, \Omega_p)$ is a sub-graph (supernode) of

G where β_p and Ω_p are the node and corridors of this sub-graph. Also, suppose that $\Phi = \{ij \in \Omega \,|\, i \in \beta_p, j \in \beta / \beta_p\}$ is a set of all lines outside sub-graph P that connect to the nodes of P. Therefore, the following equation, which is a generalization of equation (14a), is valid for the supernode P.

$$\sum_{ij\in\Phi}\sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t} \geq \left\lceil \frac{\left(\left| \sum_{i\in\beta_p} P_{i,t}^{g} - \sum_{i\in\beta_p} P_{i,t}^{load} \right| - \sum_{ij\in\Phi} N_{ij}^{0} \bar{P}_{ij} \right)}{\max\limits_{ij\in\Phi}(\bar{P}_{ij})} \right\rceil \quad \forall \mathrm{t} \in \mathrm{T} \tag{16a}$$

Example 4: In Figure 5 a supernode consisting of two buses is illustrated. The maximum number of candidate lines in this example is considered to be 1. As in previous examples, we assume that the problem is static. According to equation (16a) the following equation can be applied to the problem.

$$x_{13} + x_{14} + x_{17} + x_{25} + x_{26} \geq \left\lceil \frac{200 + 110 - 100 - 40 - 10n^{0}{}_{17} - 30n^{0}{}_{14} - 20n^{0}{}_{26}}{45} \right\rceil = 3 \tag{16b}$$

3.3.4 Constraint Type-2 in a Supernode

These types of constraints are obtained generalizing equation (15a) as well as the definition of the supernode in subsection 3.3.3.

$$\sum_{ij\in\Phi} (N_{ij}^{0} + \sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t}) \bar{P}_{ij} \geq \sum_{i\in\beta_p} P_{i,t}^{load} - \sum_{i\in\beta_p} P_{i,t}^{g} \tag{17a}$$

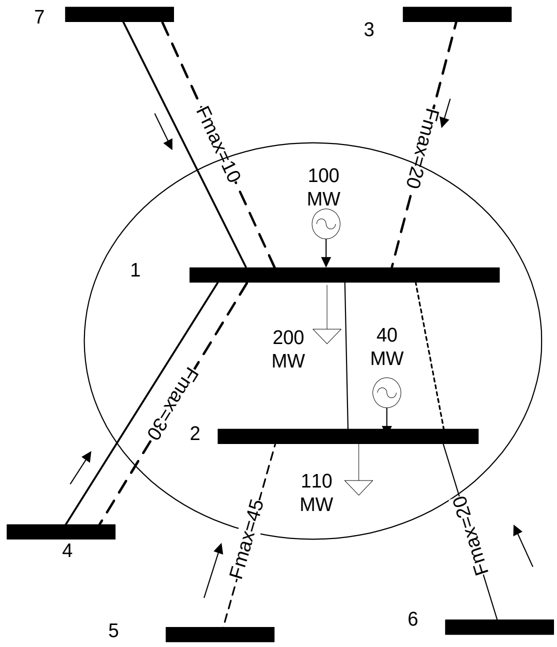
considering $L = \Phi - \{ij\}$ for some $ij \in \Phi$; therefore, more strong constraints, (17b), can be obtained for each supernode P.

$$\sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t} + \sum_{km\in L}\sum_{y=1}^{\eta_{km}} 2^{y-1} x_{km,y,t} \left\lceil \frac{\bar{P}_{km}}{\bar{P}_{ij}} \right\rceil \geq \left\lceil \frac{\sum_{i\in\beta_p} P_{k,t}^{load} - \sum_{i\in\beta_p} P_{k,t}^{g} - \sum_{km\in L} N_{km}^{0} \bar{P}_{km}}{\bar{P}_{ij}} \right\rceil - N_{ij}^{0}$$

$$\forall ij \in \Phi, \forall t \in T \tag{17b}$$

Note that constraint Type-1 in a supernode is a special case of constraint Type-2 in a supernode.

Figure 5 - A supernode consisting of two adjacent buses



Source: The author

Table 2 - The set of constraints obtained using Figure 5 and equation (17b)

	$\sum_{y=1}^{\eta_{ij}} 2^{y-1} x_{ij,y,t} + \sum_{km\in L}\sum_{y=1}^{\eta_{km}} 2^{y-1} x_{km,y,t} \left\lceil \frac{\bar{P}_{km}}{\bar{P}_{ij}} \right\rceil$					$\left\lceil \frac{\sum_{i\in\beta_p} P^{load}_{k,t} - \sum_{i\in\beta_p} P^{g}_{k,t} - \sum_{km\in L} N^0_{km}\bar{P}_{km}}{\bar{P}_{ij}} \right\rceil$	N^0_{ij}
Corridor *ij*	x_{13}	x_{14}	x_{17}	x_{25}	x_{26}		
1-3	1	$\left\lceil \frac{30}{20} \right\rceil$	$\left\lceil \frac{10}{20} \right\rceil$	$\left\lceil \frac{45}{20} \right\rceil$	$\left\lceil \frac{20}{20} \right\rceil$	$\left\lceil \frac{170-30-10-20}{20} \right\rceil$	0
1-4	$\left\lceil \frac{20}{30} \right\rceil$	1	$\left\lceil \frac{10}{30} \right\rceil$	$\left\lceil \frac{45}{30} \right\rceil$	$\left\lceil \frac{20}{30} \right\rceil$	$\left\lceil \frac{170-10-20}{30} \right\rceil$	1
1-7	$\left\lceil \frac{20}{10} \right\rceil$	$\left\lceil \frac{30}{10} \right\rceil$	1	$\left\lceil \frac{45}{10} \right\rceil$	$\left\lceil \frac{20}{10} \right\rceil$	$\left\lceil \frac{170-30-20}{10} \right\rceil$	1
2-5	$\left\lceil \frac{20}{45} \right\rceil$	$\left\lceil \frac{30}{45} \right\rceil$	$\left\lceil \frac{10}{45} \right\rceil$	1	$\left\lceil \frac{20}{45} \right\rceil$	$\left\lceil \frac{170-30-10-20}{45} \right\rceil$	0
2-6	$\left\lceil \frac{20}{20} \right\rceil$	$\left\lceil \frac{30}{20} \right\rceil$	$\left\lceil \frac{10}{20} \right\rceil$	$\left\lceil \frac{45}{20} \right\rceil$	1	$\left\lceil \frac{170-30-10}{20} \right\rceil$	1

Source: The author

Example 5: according to Figure 5 and equation (17b), a set of five constraints can be added to the problem, as shown in Table 2 and equations (17c)-(17g). Note that equations (17c) and (17g) are identical. Although the commercial solvers remove redundant constraints before solving the problem, it is good practice to remove duplicate constraints.

$$x_{13} + 2x_{14} + x_{17} + 3x_{25} + x_{26} \geq 6 \tag{17c}$$

$$x_{13} + x_{14} + x_{17} + 2x_{25} + x_{26} \geq 4 \tag{17d}$$

$$2x_{13} + 3x_{14} + x_{17} + 5x_{25} + 2x_{26} \geq 11 \tag{17e}$$

$$x_{13} + x_{14} + x_{17} + x_{25} + x_{26} \geq 2 \tag{17f}$$

$$x_{13} + 2x_{14} + x_{17} + 3x_{25} + x_{26} \geq 6 \tag{17g}$$

3.3.5 Efficiency of Fence Constraints

In subsections 3.3.1 to 3.3.4, two types of constraints have been proposed to be included in the model. In constraints type-1, (14a) and (16a), only a single constraint for each node or supernode can be created, while in constraint type-2, (15f) and (17b), several constraints, depending on the number of lines connected to that node or supernode, can be included in the model. Due to the exponential number of supernodes, a numerous number of constraints can be added to the model, resulting in a very large LP problem, which requires considerable effort to be solved at each branch and bound node. Therefore, in order to make the constraints more efficient the following considerations are taken into account:

1) The power deficit in nodes or supernodes is an indicator of the efficiency of a constraint and thus the right-hand side of constraints defines its efficiency. It is obvious that zero or a negative value on the right hand of the constraints will not help in the convergence of the problem; therefore, we have not added them to the problem.

2) A constraint can be much more efficient if the left-hand side has a fewer number of binary variables. For example, considering *x* as binary variables, $x_1 + x_2 \geq 2$ is much stronger than $x_1 + x_2 + x_3 + x_4 \geq 2$,. In the first equation, the cut can results in $x_1 = 1$ and $x_2 = 1$, but in second, there are 11 combinations of binary variables that can satisfy it. The cut in the supernode containing many buses will result in an great number of binary variables in the right hand of constraints resulting in weak constraints. Therefore, in this thesis, supernodes

of 2 and 3 buses are produced while supernodes containing more buses are not considered to be useful in the systems studied.

These two considerations will reduce the number of constraints significantly and in some systems that have a low deficit of power, such as the Colombian system discussed in the results section, leads to a small number of valid inequality constraints. Note that the calculation of power deficit in constraints (14a), (15f), (16a) and (17b) is very conservative; it was obtained assuming that all the lines connecting a node or supernode bring the power with their maximum capacity; it was also assumed that generators deliver the maximum power to the node or supernode.

3.4 SENSITIVITY INDEX

In this thesis, the sensitivity index is used in the GRASP construction phase methodology, GRASP-CP, to reduce the search space of the TEP problem. The GRASP-CP methodology will be explained in section 3.5. The purpose of a sensitivity index (SI) is to assess the impact of any integer variable that affects the performance of the problem. There are a couple of sensitivity indexes in the TEP. The first SI, proposed by Garver (1970), is calculated directly from integer variables. The Garver sensitivity index (SIG) is obtained after solving the relaxed hybrid model of the TEP problem, see (8a)-(8h). The SIG for each transmission line is calculated in (18);

$$SIG_{ij} = n_{ij}\bar{P}_{ij} \qquad \forall ij \in \Omega \tag{18}$$

Although this index is considered to be the best in terms of finding the solution that is closest to the optimum (ROMERO; ASADA et al., 2007), it is not very suitable to be used alone for the GRASP-CP. This index is too greedy, it assesses the best lines, and defines zero impact for some lines if added to the solution. This is in contradiction with the fact that every line may affect the performance of the optimum solution. There are other indexes proposed in (PEREIRA; PINTO, 1985) which are calculated from the dual value of the LP problem at the optimum solution. One of these indexes is based on the maximum flow limit and the other on circuit susceptance.

The objective of these indices is to calculate the variation of the objective function with respect to incremental variations of the system components, that is,

$$SI_i = \frac{\partial v}{\partial \mu_i} \tag{19}$$

where SI_i is the sensitivity with respect to the i^{th} system component (μ). Equation (19) can be calculated using the Lagrange multiplier (shadow prices) of a LP problem. Shadow prices evaluate the change of the objective function with respect to the change in variable μ. In this thesis, we use the Lagrange multiplier of the hybrid linear model presented in (8a)-(8h) to obtain the sensitivity index related to the installation of transmission lines. There are two ways to identify potential circuits by the Lagrange multiplier of the hybrid model:

1) SI with respect to the maximum flow limit (SIF): This index is obtained using dual variables related to constraint (8d), at the optimum solution of the LP problem and is calculated as:

$$SIF_{ij} = \frac{\partial v}{\partial \bar{P}_{ij}} = \pi_{\bar{P}_{ij}} \qquad \forall ij \in \Omega \tag{20}$$

where v is the objective function of the TM-TEP and $\pi_{\bar{P}_{ij}}$ is the dual variable associated to constraint (8d). The problem associated with this index is that only a reduced number of circuits will have flows at their limits in the optimal solution of the LP problem. Consequently, only these circuits will have an associated non-zero Lagrange multiplier Again this does not agree with experience with planning multipliers, which indicates that many of the possible additions affect system performance.

2) Sensitivity index with respect to susceptance of transmission lines:

Sensitivity analysis with respect to circuit susceptance avoids the problem of (20) and can easily be calculated from the optimal solution of the LP problem. It is shown in (DECHAMPS; JAMOULLE, 1980) that the sensitivity index with respect to variation of transmission lines susceptance can be expressed as:

$$SIY_{ij} = \left[\pi_d(i) - \pi_d(j)\right](\theta_i^* - \theta_j^*) \tag{21}$$

where $\pi_d(i)$ and $\pi_d(j)$ are Lagrange multipliers with respect to constraint (8b) at the optimum solution of LP problem. θ_i^* and θ_j^* are bus voltage angles at buses *i* and *j*.

In (BINATO et al., 2001a), it is claimed that this index is usually negative, indicating a marginal benefit of adding a new line to the network. They also considered the investment cost of transmission lines in order to gain a better assessment of the potential lines for addition. The sensitivity index proposed by them is given in (22).

$$SIH_{ij} = \frac{SIY_{ij}}{C_{ij}} \tag{22}$$

Sensitivity indices in ((18), (20) and in (22)) have different characteristics and potential for identifying the effect of a line in a solution. Therefore, a combination of three sensitivity indices ((18), (20) and in (22)), will result in better performance. For each line, the sensitivity index of (23) is calculated, where the *SIG*, *SIF* and *SIH* are normalized in order to have equal effects in the total sensitivity index (SI).

$$SI_{ij} = \frac{SIG_{ij}}{\max(SIG_{km\in\Omega})} + \frac{SIF_{ij}}{\max(SIF_{km\in\Omega})} + \frac{SIH_{ij}}{\max(SIH_{km\in\Omega})} \qquad \forall ij \in \Omega \tag{23}$$

3.5 REDUCTION OF SEARCH SPACE USING THE GRASP CONSTRUCTION PHASE

The branch and bound algorithm can be employed to solve the reduced disjunctive model formulated in (13a)–(13l) together with the fence constraints discussed in 3.3, i.e., (14a), (15f), (16a) and (17b). In large-scale systems, the number of binary decision variables is very large; as a result, the time needed for obtaining a solution grows exponentially and the branch and bound is unable to converge to the optimum solution or at least high quality solutions. Therefore, in this thesis, the search space of binary variables is reduced using GRASP-CP. However, it should be observed that, unlike methods discussed in previous sections, the GRASP-CP may exclude the optimum point of the problem.

A complete description of GRASP can be found in (BINATO et al., 2001b; FARIA et al., 2005; FEO; RESENDE, 1995). GRASP is a heuristic iterative sampling technique composed of two phases: a construction phase and a local search phase. In this thesis, only the construction phase of the GRASP is used since the local search phase is very time consuming and may tighten the search space by converging to local solutions. In the construction phase, a feasible solution is iteratively constructed, one element at a time. At each construction iteration, another element to be included is selected randomly from a restricted candidate list (RCL). The RCL is constructed from elements with a greedy function value above a specified threshold. The greedy function value of an element is evaluated by measuring the local benefit of including that element in the constructed solution. The random selection of an element from a restricted candidate list, constructed by a greedy function, characterizes both the randomness and greediness of the GRASP-CP procedure. The generic pseudo code of the GRASP-CP procedure for creating a feasible solution ($\chi_{ij\in\Omega}$) for a static TEP problem is shown in Figure 6.

Figure 6 - Pseudo code of GRASP-CP for the static TEP problem using the hybrid model

Procedure GRASP-CP
Result: solution ($\chi_{ij\in\Omega}$)
Data: RCL parameter (α)
1 $\chi_{ij\in\Omega} = 0$
2 **Repeat**
3 $n_{ij\in\Omega} \leftarrow$ Solve_Relaxed_HLM_Model ($\chi_{ij\in\Omega}$)
4 $SI_{ij\in\Omega} \leftarrow$ equation (23)
5 $SI_{\min} = \min\{SI_{ij} \mid ij \in \Omega\}$
6 $SI_{\max} = \max\{SI_{ij} \mid ij \in \Omega\}$
7 $RCL = \{ij \mid SI_{ij\in\Omega} \geq SI_{\min} + \alpha(SI_{\max} - SI_{\min})$
8 select $ij \in RCL$ at random
9 $\chi_{ij} = \chi_{ij} + 1$
10 **Until** a feasible solution is obtained ($SI_{\max} > 0$)
Return with feasible solution

Source: The author

In GRASP-CP procedure, a restricted candidate list parameter (α) is provided by the planner in the interval [0,1]. This value is not changed during the construction phase and defines the greediness or randomness of the RCL set. If $\alpha = 1$, then the semi-greedy construction phase reduces to a greedy algorithm, and if $\alpha = 0$, it changes to a random algorithm.

In Figure 6, the solution ($\chi_{ij\in\Omega}$) is initialized to zero (row 1) and in rows 2 to 10, it is gradually constructed in an iterative process. In each iteration, a transmission line is added to the solution. In rows 3 to 9, a transmission line is added to the solution through a semi-greedy procedure. In row 3, the integer variables of the HLM model are relaxed and the resulting LP is solved considering the already constructed solution ($\chi_{ij\in\Omega}$) added to the existing transmission lines N_{ij}^{0}, i.e., replacing N_{ij}^{0} with $N_{ij}^{0}+\chi_{ij}$ in the HLM model. In row 4, the sensitivity index is calculated based on equation (23). This index is capable of identifying a potential transmission line for insertion into the solution. However, with respect to GRASP-CP methodology, we do not insert the best transmission line into the solution. For this reason, in rows 5 and 6, the minimum and maximum value of the sensitivity index is identified and then the restricted candidate list is created with candidate transmission lines that have a sensitivity index value above a specified threshold value as stated in row 7. Finally, in rows 8 and 9, a transmission line is randomly selected to be included into the solution. The inclusion of this line in the solution alters the semi greedy threshold and the set of candidate transmission lines used to determine the next RCL. The line addition is continued until a feasible solution is obtained. When the sensitivity index becomes zero, it means that there is no need to add more candidate lines into the solution and the process stops. It should be noted that the solution obtained by this algorithm is feasible for the DC model, since those candidate lines considered for addition in previous iterations are treated as existing lines in the current iteration, in which both Kirchhoff's laws are considered.

The procedure of GRASP-CP, shown in Figure 6, constructs a single solution. However, to reduce the search space of the problem, several different solutions are needed. Therefore the procedure shown in Figure 7 is proposed. This procedure uses the procedure of Figure 6, GRASP-CP, several times as a subroutine to create several different feasible high-quality solutions. The best solution ($\chi^{*}{}_{ij\in\Omega}$) over all iterations is considered to be the initial incumbent solution of the branch and bound algorithm. The maximum value of all solutions in each corridor is set as an upper limit of candidate transmission lines (n_{ij}) in that corridor.

In the pseudo code of domain reduction of the TEP problem using GRASP-CP, the number of calls to GRASP-CP (i_{max}) and the parameter α are predefined data. In row 1 of the pseudo code, the objective of the incumbent solution is set to infinity and, in row 2, the set of all solutions ($\sigma_{ij\in\Omega}^{k}$, where k is the solution number) is initialized at zero. In row 4, with a call to GRASP-CP, we get a solution to the problem. The cost of this solution (v) is obtained in row 5, and, in row 6, it is compared with the best solution of the previous iterations. If the investment is less, it will be considered the incumbent solution in rows 7 and 8. In each iteration, the k^{th} solution is reserved as $\sigma_{ij\in\Omega}^{k}$ and will be used later in line 11 to form the upper bound of binary variables, i.e., the maximum number of transmission lines in a corridor. As a result, the search space will be reduced significantly.

Figure 7 - Pseudo code of domain reduction of the TEP problem using GRASP-CP

Procedure: Domain Reduction using GRASP-CP
Result: Incumbent solution ($\chi^{*}_{ij\in\Omega}$), new upper bound for integer variables ($\overline{N}_{ij}$)
Data: Number of iterations (k_{max}), RCL parameter (α)

$v^{*} = \infty$
$\sigma_{ij}^{k} = 0 \ \forall ij \in \Omega, \forall k \in \{1..k_{\max}\}$
for $k = 1, \ldots, k_{max}$ **do**
  $\sigma^{k}_{ij\in\Omega} \leftarrow$ GRASP-CP
  $v = \sum_{ij\in\Omega} C_{ij}\sigma_{ij}^{k}$
  if $v < v^{*}$ **then**
    $v^{*} \leftarrow v$
    $\chi^{*}_{ij\in\Omega} \leftarrow \sigma^{k}_{ij\in\Omega}$
  end
end
$\overline{N}_{ij} = \max\{\sigma^{k}_{ij} \mid k \in 1..k_{\max}, ij \in \Omega\}$
Return

Source: The author

3.5.1 Domain Reduction Using GRASP-CP an Example

An example is provided in this section to show how GRASP-CP reduces the search space of a TEP problem. The Garver 6-bus system (Figure 8) is used for this purpose. This system has 6 buses and 15 candidate lines with a total demand of 760 MW and a maximum of 5 lines can be added to each corridor. The system data is given in (ROMERO et al., 2002) and also in Appendix B. II. The optimum solution without generation rescheduling for the DC model is reported in (ROMERO et al., 2002), with a US$200 investment cost on seven new transmission lines with the following topology: $n_{2\text{-}6} = 4$, $n_{3\text{-}5} = 1$ and $n_{4\text{-}6} = 2$.

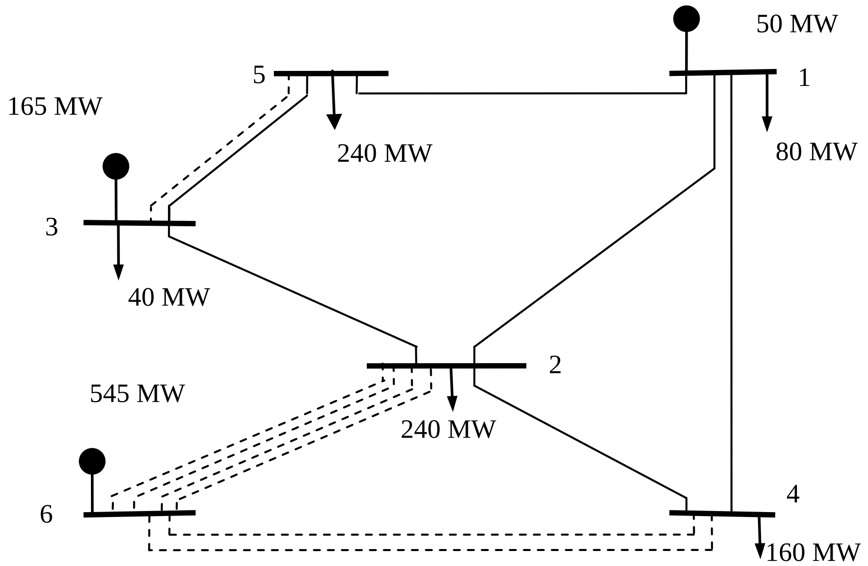
The GRASP-CP, shown in Figure 6 and Figure 7, with $\alpha = 0.5$ is employed to reduce the domain of integer variables. As shown in Table 3, initially 10 solutions were created, each one different, and then, the maximum of each element is calculated to create the reduced search space. The optimum solution is also shown in the rightmost column of Table 3. From the information in this table it can be verified that the optimum solution of the Garver system is within the reduced search space of the problem. This fact is also shown graphically in Figure 9. In this figure, the black bars show the reduced space of the problem while the gray bars show the optimum solution of the problem. The integer variables are limited by black bars. This figure shows that the domain of integer variables is reduced without losing the optimum solution. As indicated above, the maximum number of lines in each corridor in the Garver system is 5, multiplying this number by the number of candidate lines, gives 75 as the total bounds of integer variables. When using GRASP-CP, this number is 11, demonstrating that the upper bounds of integer variables is reduced by 85.33 %. The number of combinations in the full space of the problem is $6^{15} \approx 4.70 \times 2^{11}$ while the number of combinations using GRASP-CP is dramatically reduced to 540 (2×5×2×2×3×3).

Table 3 - GRASP-CP procedure for reducing the search space of the DC model of the TEP problem in the Garver system

Corridor	Solutions Obtained										Max	Opt.
From-to	1	2	3	4	5	6	7	8	9	10	---	---
1-2	0	0	0	0	0	0	0	0	0	0	0	0
1-3	0	0	0	0	0	0	0	0	0	0	0	0
1-4	0	0	0	0	0	0	0	0	0	0	0	0
1-5	0	0	0	0	0	0	0	0	0	0	0	0
1-6	1	0	1	1	1	1	0	1	0	1	1	0
2-3	0	0	0	0	0	0	0	0	0	0	0	0
2-4	0	0	0	0	0	0	0	0	0	0	0	0
2-5	0	0	0	0	0	0	0	0	0	0	0	0
2-6	2	3	2	3	4	3	3	3	3	4	4	4
3-4	0	0	0	0	0	0	0	0	0	0	0	0
3-5	0	1	0	1	1	1	1	1	0	0	1	1
3-6	0	1	0	1	0	0	0	1	0	0	1	0
4-5	0	0	0	0	0	0	0	0	0	0	0	0
4-6	2	2	2	2	2	2	2	2	2	2	2	2
5-6	2	1	2	1	0	1	1	0	2	2	2	0
Cost	31	27	31	34	26	29	28	29	27	37	---	200
(US$)	0	9	0	7	8	9	6	2	2	0		

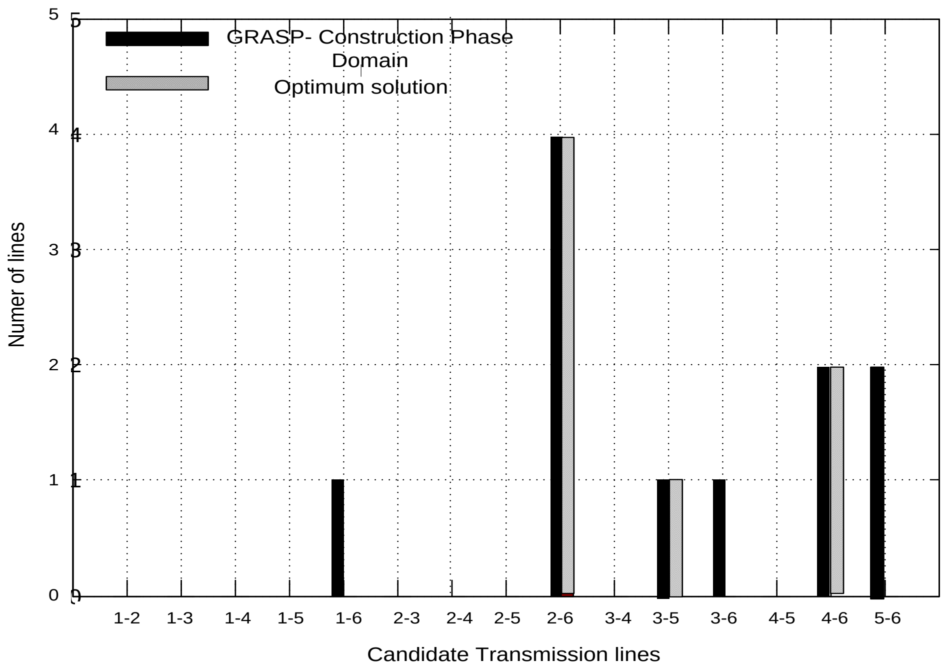
Source: The author

Figure 8 - The Garver system with the base case topology and optimum solution



Source: The author

Figure 9 - GRASP-CP for reducing the domain of integer variables in the Garver system (DC model)



Source: The author

MULTISTAGE TRANSMISSION EXPANSION PLANNING CONSIDERING FIXED SERIES COMPENSATION ALLOCATION

4.1 INTRODUCTION

Electricity demand is increasing much faster than transmission system growth (U.S. DEPARTMENT OF ENERGY, 2008). In many countries, building new transmission lines or upgrading existing ones is extremely difficult due to geographical, environmental or other related concerns, and the process may take several years to be completed (ABB AB FACTS, 2010). As a result, there has been chronic underinvestment in transmission lines, jeopardizing system reliability (U.S. DEPARTMENT OF ENERGY, 2008).

In recent years there has been considerable interest in localizing fixed series compensation (FSC) for better use of the existing grid, delaying the construction of new lines or temporarily removing system bottlenecks. However, the inclusion of FSCs in new transmission lines is rare due to different design lifetimes and also because it is usually considered as a short-term solution for power system problems. In (ABB AB FACTS, 2010) it is demonstrated that including FSCs in new transmission lines can eliminate the necessity of installing many parallel transmission lines and therefore can significantly reduce the total investment cost.

FSC can be integrated in transmission expansion planning because some power lines are frequently overloaded while, at the same time, other power lines have remaining capacity. By using series compensation, it is possible to redistribute the power flow to use this capacity. The use of FSCs in transmission planning results in a different grid topology at much lower cost due to better utilization of the whole transfer capacity of the network, meaning that, with a little investment in FSCs, a significant benefit-cost ratio can be achieved.

Redistribution of power is possible with several flexible AC transmission systems (FACTS) such as thyristor controlled series capacitor (TCSC), static synchronous series compensator (SSSC), unified power flow controller (UPFC), and interline power flow controller (IPFC). However, the simplest and most cost-effective transmission technology suitable for redistributing power can be provided by FSC (SIEMENSE ENERGY, 2013).

There are also many other objectives for using FSCs including improvement of power quality, minimization of power losses, voltage and transient stability enhancement, and transfer capability enhancement (ANDERSON, 1996; CHANG; YANG, 2001; GRÜNBAUM et al., 2010; KIMBARK, 1966; LEONIDAKI et al., 2005; MILLER, 1982; ZAMORA-CÁRDENAS; FUERTE-ESQUIVEL, 2010).

The (N-1) security constraints need to be considered in transmission expansion planning to ensure the viable operation of the system in case of contingencies (SILVA et al., 2005). This criterion states that the system should be reinforced to operate adequately if a line was withdrawn or an FSC bypassed.

In this thesis, FSCs are proposed for integration in the multistage transmission expansion planning (TEP) problem when the planning horizon is divided into several stages (ESCOBAR et al., 2004). In multistage planning, FSC can be included in the system to remove system bottlenecks in certain planning stages and can be renewed, replaced or even excluded from the system in later stages as a result of the lifetime design or operational considerations. In this thesis, FSCs are considered for integration in future transmission lines in order to lower the necessity of constructing many transmission lines and significantly decrease investment costs and construction time. A new mixed binary linear model for the multistage expansion of transmission systems considering the allocation of FSC with N-1 security constraints for both transmission lines and FSC is proposed and solved using a branch and bound algorithm.

The aims of solving the multistage TEP problem considering FSC allocation are to define 1) compensated transmission corridors, 2) series compensation percentages, and 3) reinforcement transmission lines in each planning stage. At the same time, the (N-1) security constraints are also included to ensure the robust operation of the system for predefined contingencies.

This chapter is organized as follows. Section 4.2 provides the structure, advantages, disadvantages, and investment costs of FSCs in a power system. Section 4.3.2 discusses technical issues related to the insertion of FSCs in a multistage TEP. In section 4.3, the mathematical modeling of the multistage TEP with allocation of FSCs and considering security constraints is provided.

4.2 FIXED SERIES COMPENSATION

4.2.1 Advantages and Drawbacks

Series compensation reduces the natural reactance of transmission lines and provides the following benefits for the power system (ANDERSON, 1996; KIMBARK, 1966; MILLER, 1982):

1. It increases the power transfer capacity of transmission corridors, improves load sharing among transmission lines and thus reduces transmission bottlenecks.

2. It improves system stability and voltage regulation and reduces voltage variation.

3. It can be installed in existing substations, without acquisitions of new corridors.

Regardless of the benefits of using FSC, some drawbacks must be considered:

1. Resonance is possible with thermal generators close to a series-compensated transmission line. This is the main restriction for FSC allocation in a system (ALIZADEH-PAHLAVANI; MOHAMMADPOUR, 2011; GRÜNBAUM et al., 2010; KIMBARK, 1966).

2. FSC may be exposed to overvoltages from different system faults, with the most dangerous resulting from internal and external transmission lines faults.

3. Careful design is required for substations with high fault currents (40 kA and higher).

However, these problems have feasible solutions; and with technological advances in protection system, there is no fear of inserting series compensation in a transmission system (ANDERSON, 1998). Anderson (1998) states that:

“Even though the series compensation has been known to create problems in system protection and subsynchronous resonance, the return is usually considered worth the extra engineering effort required to properly design and operate these more complex transmission facilities”.

4.2.2 Investment Cost

The investment cost for FSC equipment involves many factors: market competition, local legislation, taxation, environmental constraints, etc. Normally, in the early stages of an FSC project, the manufacturers provide only global prices for the compensation project (K_s) in US$/Mvar. The cost of the protection system is usually considered in the FSC global price. Therefore, the investment cost per km for FSC with Q_s Mvar transfer capacity is calculated by $K_s Q_s$. Q_s can be defined as a percentage (ρ_S) of the maximum transfer capacity of a transmission line ($Q_{\max}$) (ANDERSON, 1996; KIMBARK, 1966; MILLER, 1982):

$$Q_s = \rho_S Q_{\max} = 3\rho_S I_{\max}^2 X_l \tag{24}$$

where $I_{\max}$ is the maximum line current in kA and X_l is transmission line reactance in ohm/km. To include FSC in the TEP problem, one can estimate the investment ratio of FSC with respect to the transmission line as stated by (25).

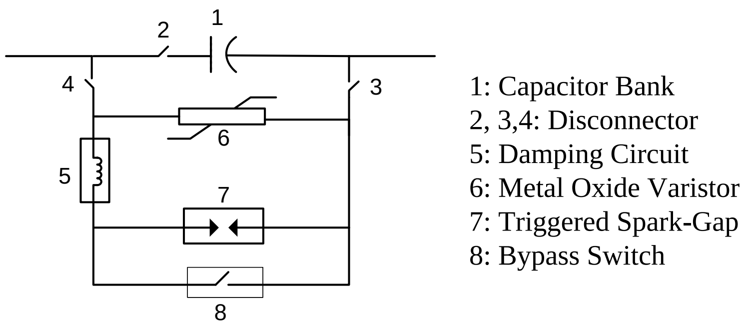
$$C_s = \frac{\text{FSC invesment cost per km}}{\text{Line invesment cost per km}} = \frac{K_s Q_s}{K_l} = \frac{3K_s \rho_S I_{\max}^2 X_l}{K_l} \tag{25}$$

For standard voltage levels (115 kV, 230 kV, 500 kV), $I_{\max}$, X_l, K_l and K_s are standard quantities; thus, C_s depends only on the percentage of compensation (ρ_s). Therefore, in the TEP problem with FSC allocation, by using the C_s factor, it is possible to find the FSC investment cost based on the transmission line investment cost and the desired FSC percentage. For example, for a transmission line at 115 kV with certain characteristics (ρ_S =20 %, K_s = 60000 US$/Mvar, $I_{\max}$ = 0.6 kA, K_l = 98000 US$/km and X_l = 0.52 ohm/km), the FSC investment factor (C_s) equals 6.8 %.

4.2.3 Structure

Figure 10 shows a simplified structure of a series capacitor with its main components. The FSC is made up of the actual capacitor banks, metal oxide varistors (MOVs), damper circuit, triggered spark-gap, bypass switch and three high voltage switches (ABB AB FACTS, 2010; DE OLIVEIRA, 2008). The MOVs protect the Capacitor Banks from overvoltages during and after transmission system failures and must be designed to withstand the high fault current until bypass occurs. The triggered spark-gap protects the MOVs against excessive energy absorption. The bypass switch is used for intentional or emergency bypassing of the capacitor banks for some specified time or continuously. Three high-voltage disconnecting switches serve to integrate the FSC and isolate it from the transmission line. The damping circuit limits and damps the discharge current caused by spark gap operation or closing the bypass switch. The IEEE draft guide for the functional specification of fixed transmission series capacitor banks for transmission system applications contain detailed guidelines for FSC protection system.

Figure 10 - Simplified structure of a fixed series capacitor



Source: The author

4.3 MATHEMATICAL MODELING

As indicated above, the main effect of FSC in TEP is redistributing the power flow by modifying the natural reactance of the transmission lines. Thus, the DC model seems suitable for modeling this problem. In TEP with allocation of FSCs, both in static and multistage planning, the following considerations are taken into account:

1) FSC is allocated to balance the load sharing between parallel transmission lines. Therefore, identical transmission lines have equal amounts of compensation.

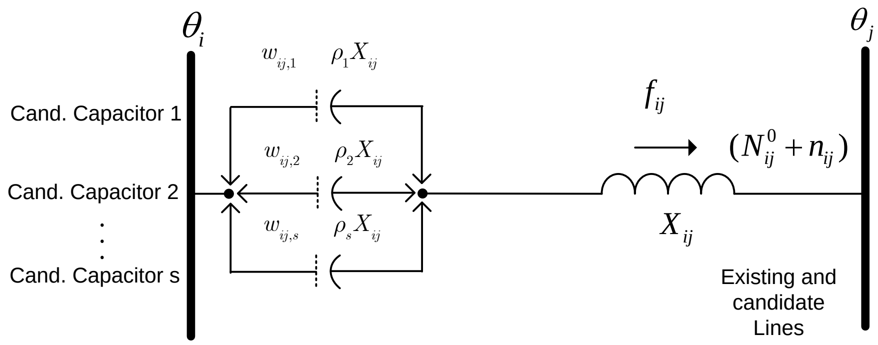
2) FSC is not allocated in short transmission lines. FSC has been found to be economically justified when allocated in transmission lines longer than 200 km (ABB, 2012).

The complete mathematical model of multistage transmission expansion planning considering allocation of fixed series compensation and N-1 security constraints has many indices, variables and constraints which initially make it difficult to understand. Therefore, in this section, we first provide the nonlinear mathematical model for the static planning problem without security constraints. Then, the complete model is provided in subsection 4.3.2.

4.3.1 Nonlinear Static Model of TEP with FSC without Security Constraints

Figure 11 is provided to model transmission lines and FSCs between two system buses, *i* and *j*. In this figure, N_{ij}^0 and n_{ij} are used to represent the existing and candidate transmission lines. There are *s* types of FSCs, each with different compensation rates that can be allocated in each existing or candidate line. For each line, just one of the candidate FSCs can be installed. The number of candidate FSCs depends on the corridor and the transmission system and is defined prior to planning. The installation of FSCs in existing and candidate lines is shown using a set of binary variables $w_{ij,s}$. If $w_{ij,s}=1$, all the existing and candidate lines of similar type are compensated with ρ_s compensation percentage.

Figure 11 - General model for a transmission corridor with existing lines, candidate lines and FSCs in a static model of TEP with FSC



Source: The author

Considering Figure 11 the mixed integer nonlinear static DC model of static TEP without security constraints, considering the allocation of FSC programming problem, is given in (26a)-(26k).

FSCSTEP: Min $v = \sum_{ij\in\Omega} C_{ij} n_{ij} + \sum_{ij\in\Omega}\sum_{s\in S} C_{ij} C_s (N_{ij}^0 + n_{ij}) w_{ij,s}$ (26a)

Subject to:

$$p_i^g - \sum_{ij \in \Omega} f_{ij} + \sum_{ji \in \Omega} (f_{ji}) = P_i^{load} \qquad \forall i \in \beta \tag{26b}$$

$$f_{ij} X_{ij} (1 - \sum_{s \in S} \rho_s w_{ij,s}) = (n_{ij} + N_{ij}^0)\theta_{ij} \qquad \forall ij \in \Omega \tag{26c}$$

$$\left| f_{ij} \right| \le (n_{ij} + N_{ij}^0)\bar{P}_{ij} \qquad \forall ij \in \Omega \tag{26d}$$

$$\sum_{s \in S} w_{ij,s} \le 1 \qquad \forall ij \in \Omega \tag{26e}$$

$$\sum_{s \in S} \rho_s w_{\mathrm{ij},s} \le \bar{\rho}_s \qquad \forall ij \in \Omega \tag{26f}$$

$$w_{ij,s} - N_{ij}^0 - n_{ij} \le 0 \qquad \forall ij \in \Omega, \forall s \in S \tag{26g}$$

$$\left|\theta_{ij}\right| \le \bar{\theta} \qquad if \;\; (N_{ij}^0 + n_{ij}) \ge 1 \qquad \forall ij \in \Omega \tag{26h}$$

$$0 \le n_{ij} \le \bar{N}_{ij} \qquad \forall ij \in \Omega \tag{26i}$$

$$w_{ij,s} \text{ binary} \qquad \forall ij \in \Omega, \forall s \in S \tag{26j}$$

$$n_{ij} \text{ integer} \qquad \forall ij \in \Omega \tag{26k}$$

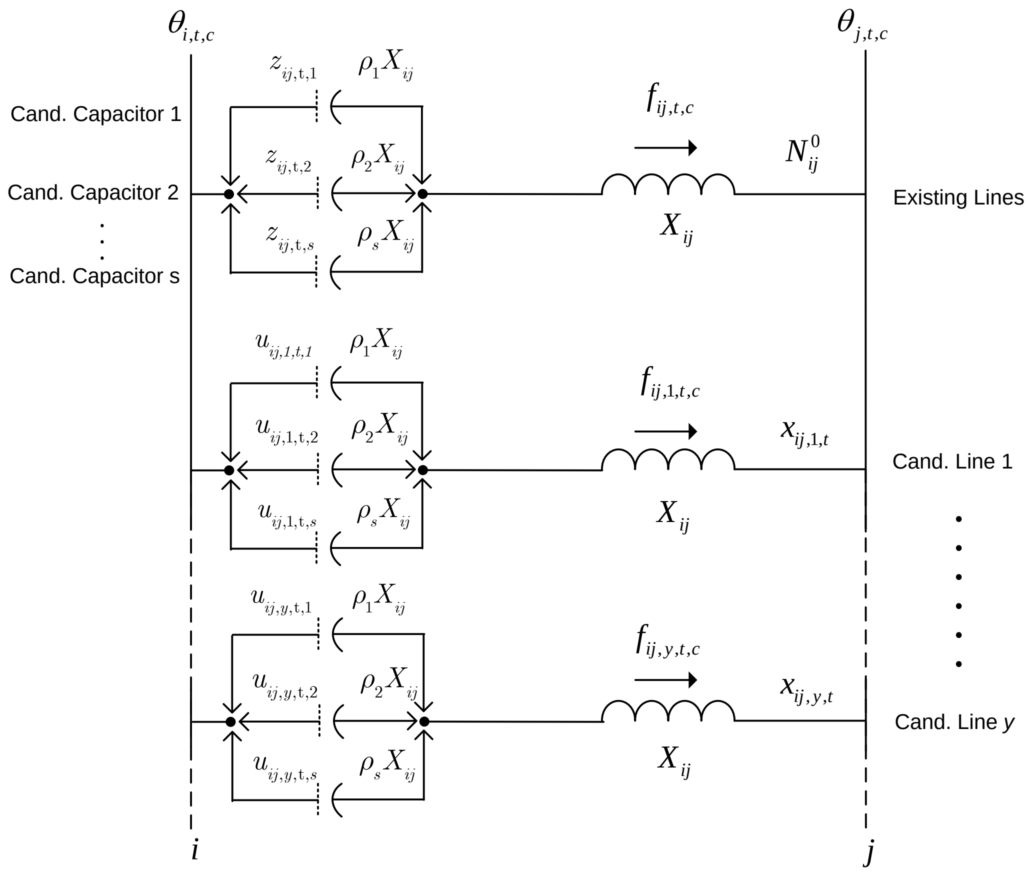
Objective function (26a) considers the investment costs in the reinforcement lines and in FSCs. The investment for each FSC type is calculated using the C_s factor and reinforcement line costs, according to (25). Note that for each transmission corridor, the number of series compensators allocated is equal to the number of existing and reinforcement lines in that corridor. Constraint (26b) represents power balances in each node. The power flows through the existing and reinforcement transmission lines are calculated in (26c). Constraint (26d) limit the active power flows in the transmission lines to a maximum value of $\bar{P}_{ij}$.Constraint (26e) prevents superposition in the FSC allocation so that it is possible to install only one FSC type per transmission corridor. The maximum FSC percentage for each corridor is given by (26f). To ensure installation of FSC only in existing and reinforcement transmission lines, constraint (26g) is considered. This restriction avoids exploring a solution that allocates FSC in corridors without existing transmission lines. Constraint (26h) is the maximum voltage angle difference allowed in each corridor and represents a stability constraint. The maximum number of reinforcement transmission lines is represented by (26i). The binary and integer nature of the investment variables is stated by (26j) and (26k).The objective function (26a) and constraints expressed by (26c) and (26h) are nonlinear because they contain the products of two variables.

4.3.2 Multistage TEP with allocation of FSC under security constraints

4.3.2.1 Structure and assumptions for allocation of FSCs in the TEP problem

Figure 12 models transmission lines and FSCs between two system buses in stage *t*, contingency *c*, and corridor *ij*. This kind of modeling is necessary to obtain a linear model. The candidate lines in stage *t* and corridor *ij* are shown using the set of binary variables $x_{ij,y,t}$ where index *y* shows the y^{th} line in this corridor and varies between 1 to $\overline{N}_{ij}$.

Figure 12 - General model for a transmission corridor with existing lines, candidate lines and FSCs



Source: The author

The installation of FSCs in existing and candidate lines is shown using the set of binary variables $z_{ij,s}^{t}$ and $u_{ij,y,s}^{t}$, respectively. If $z_{ij,s}^{t} = 1$ all the existing lines of similar type are compensated with ρ_s compensation percentage, and if $u_{ij,y,t,s} = 1$ the y^{th} candidate line is compensated again with ρ_s compensation percentage.

The following considerations are taken into account in the modeling of the multistage TEP with the allocation of FSCs in the presence of security constraints:

1) The multistage planning problem is composed of a number of static planning problems (planning stages) with their investment variables concatenated; the objective function minimizes the net present value of the investment. Thus, in multistage problems, the number of continuous and binary variables and the number of constraints linearly increase with respect to the number of stages.

2) When an outage occurs in a transmission line or FSC, the values of network operation variables, such as voltage angles and power flows, change to adapt to the new system conditions or to satisfy Kirchhoff's laws. Because the values of the network variables are different for normal and contingency conditions, we need different network variables to model them. Therefore, the number of contingency dependent variables and constraints, linearly increases with respect to the number of outages.

3) The maximum power flow in transmission lines in the contingency cases is assumed to be slightly higher than at normal levels of operation. This thesis considers a 20% increase in the line power flow. A more realistic value can be applied according to the transmission line properties (INTELLIGRID ARCHITECTURE, 2004). However, generation levels remain unchanged because they cannot change their output immediately (KAZEROONI; MUTALE, 2010).

4) It is assumed that the FSC protection system can protect the system against any internal or external faults. In case of a severe fault in the system, the protection system may bypass FSC. Therefore, the expanded network needs to be prepared for such conditions. In this thesis, we propose a model that guarantees system operation after bypassing an FSC. That is to say, the transmission system is robust in N-1 contingency in FSCs.

5) There are a number of successful protections against Sub-Synchronous Resonance (SSR) (ALIZADEH PAHLAVANI; MOHAMMADPOUR, 2011; GRÜNBAUM et al., 2010). In the planning stage, it may be difficult to consider sophisticated

countermeasures against SSR. However, this thesis considers two simple countermeasures for SSR. The first countermeasure is to install an FSC compensation percentage (ρ_S) no greater than 70%, which is usually considered safe to avoid SSR in many cases (ANDERSON, 1998). The second simple countermeasure is to bypass a portion or the whole capacitor to avoid SSR (ANDERSON, 1998). In this thesis, the transmission system is expanded in such a way that if a portion or, in worst case scenario, the whole FSC were withdrawn from the system, then the network would still work properly.

6) It is not economical to remove a device once it is installed. Therefore, devices installed in one stage are operative in the next stages; that is, planning stages are dependent on each other through investment variables.

4.3.2.2 Linear multistage model

In this section the mathematical model of multistage TEP considering the allocation of FSCs under (N-1) security constraints is proposed. First, the whole model is provided and then it is explained in three sections: a) objective function by (27a), b) contingency independent constraint, (27b)-(27p), which mainly applies to the investment variables, and c) contingency dependent constraints (27q)-(27mm), which usually contain network constraints. Note that the operation variables are indexed by the system conditions (C) and planning stages (T) while the investment variables are indexed only by T. In order to implement contingency conditions in the formulation, the parameter E_{ij}^{c} is used. This parameter states the status of a component in the system. If $E_{ij}^{c} = 0$ that indicates that a component in corridor *ij* and contingency condition *c* is in normal operation. If $E_{ij}^{c} = 1$ it means that the outage of a component is anticipated for that component in corridor *ij* and condition *c*. There are four conditions: normal system condition (C^0), contingencies in existing lines (C^1), contingencies in candidate lines (C^2) and contingency in FSCs (C^3) .

FSCMTEP:

$$Min\ v=\lambda_1\sum_{ij\in\Omega}C_{ij}\sum_{y\in Y}x_{ij,y,t}+\sum_{t=2}^{T}\lambda_t\sum_{ij\in\Omega}C_{ij}\left[\sum_{y\in Y}(\mathrm{x}_{ij,y,t}-x_{ij,y,t-1})\right]+\lambda_1\sum_{ij\in\Omega}C_{ij}\left(\sum_{s\in S}C_s(z_{ij,1,s}N_{ij}^0+\sum_{y\in Y}u_{ij,1,y})\right)+$$
$$\sum_{t=2}^{T}\lambda_t\sum_{ij\in\Omega}C_{ij}\left[\sum_{s\in S}C_s\left((z_{ij,\mathrm{t},s}-z_{ij,\mathrm{t}-1,s})N_{ij}^0+\sum_{y\in Y}(u_{ij,\mathrm{t},y}-u_{ij,\mathrm{t}-1,y})\right)\right] \quad (27a)$$

s.t.

$$0\le p_{i,t}^g\le\overline{P}_{i,t}^g \qquad \forall i\in\beta,\forall t\in T \quad (27b)$$

$$z_{ij,\mathrm{t},s},u_{ij,y,\mathrm{t},s},x_{ij,y,t}\in\{0,1\} \qquad \forall ij\in\Omega,\forall y\in Y,\forall s\in S,\forall t\in T \quad (27c)$$

$$\sum_{s\in S}z_{ij,\mathrm{t},s}\le 1 \qquad \forall ij\in\Omega,\forall t\in T \quad (27d)$$

$$\sum_{s\in S}u_{ij,y,\mathrm{t},s}\le 1 \qquad \forall ij\in\Omega,\forall y\in Y,\forall t\in T \quad (27e)$$

$$\sum_{s\in S}\rho_s z_{ij,t,s}\le\overline{\rho}_s \qquad \forall ij\in\Omega,\forall t\in T \quad (27f)$$

$$\sum_{s\in S}\rho_s u_{ij,y,t,s}\le\overline{\rho}_s \qquad \forall ij\in\Omega,\forall y\in Y,\forall t\in T \quad (27g)$$

$$\sum_{s\in S}z_{ij,t,s}-N_{ij}^0\le 0 \qquad \forall ij\in\Omega,\forall t\in T \quad (27h)$$

$$\sum_{s\in S}u_{ij,y,\mathrm{t},s}-x_{ij,y.t}\le 0 \qquad \forall ij\in\Omega,\forall y\in Y,\forall t\in T \quad (27i)$$

$$\left|z_{ij,t,s}-u_{ij,y,\mathrm{t},s}\right|\le 1-x_{ij,y,t} \qquad \forall ij\in\Omega\,|\,N_{ij}^0\ge 1,y=1,\forall s\in S,\forall t\in T \quad (27j)$$

$$\left|u_{ij,y-1,\mathrm{t},s}-u_{ij,y,t,s}\right|\le 1-x_{ij,y,t} \qquad \forall ij\in\Omega,\forall y\in Y,y>1,\forall s\in S,\forall t\in T \quad (27k)$$

$$z_{ij,\mathrm{t},s}-z_{ij,t-1,s}\ge 0 \qquad \forall ij\in\Omega,\forall y\in Y,\forall t\in T\,|\,t>1 \quad (27l)$$

$$u_{ij,y,\mathrm{t},s}-u_{ij,y,\mathrm{t}-1,s}\ge 0 \qquad \forall ij\in\Omega,\forall y\in Y,\forall s\in S,\forall t\in T\,|\,t>1 \quad (27m)$$

$$\sum_{y\in Y}x_{ij,y,t}\le\overline{N}_{ij} \qquad \forall ij\in\Omega,\forall t\in T \quad (27n)$$

$$x_{ij,y-1,t}-x_{ij,y,t}\ge 0 \qquad \forall ij\in\Omega,\forall y\in Y\,|\,y>1,\forall t\in T \quad (27o)$$

$$x_{ij,y,t}-x_{ij,y,t-1}\ge 0 \qquad \forall ij\in\Omega,\forall y\in Y,\forall t\in T\,|\,t>1 \quad (27p)$$

$$P_{i,t}^g-\sum_{ij\in\Omega}(f_{ij,t,c}^0+\sum_{y\in Y}f_{ij,y,t,c})+\sum_{ji\in\Omega}(f_{ji,t,c}^0+\sum_{y\in Y}f_{ji,y,t,c})=P_{k,t}^{load} \qquad \forall i\in\beta,\forall t\in T,\forall c\in C \quad (27q)$$

$$f^0_{ij,t,c} X_{ij} - \sum_{s \in S} \psi^0_{ij,t,s,c} = N^0_{ij} \theta_{ij,t,c} \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^0 \cup C^2 \cup C^3 \qquad (27r)$$

$$\left| \frac{\psi^0_{ij,t,s,c}}{X_{km} \rho_s} \right| \le N^0_{ij} \overline{P}_{ij,c} z_{ij,t,s} \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^0 \cup C^2 \qquad (27s)$$

$$\left| f^0_{ij,t,c} - \frac{f^0_{ij,t,s,c}}{X_{km} \rho_s} \right| \le N^0_{ij} \overline{P}_{ij,c} (1 - z_{ij,t,s}) \qquad \forall ij \in \Omega_d, \forall t \in T, \forall c \in C^0 \cup C^2 \qquad (27t)$$

$$\left| \frac{\psi f^0_{ij,t,s,c}}{X_{ij} \rho_s} \right| \le N^0_{ij} (1 - E_{ij,c}) \overline{P}_{ij,c} \; z_{ij,s} \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^3 \qquad (27u)$$

$$\left| f^0_{ij,t,c} - \frac{\psi^0_{ij,t,c,s}}{X_{ij} \rho_s} \right| \le N^0_{ij} \overline{P}_{ij,c} \left(1 - z_{ij,t,s} (1 - E_{ij,c}) \right) \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^3 \qquad (27v)$$

$$f^0_{ij,t,c} X_{ij} - \sum_{s \in S} \psi^0_{ij,t,c,s} = (N^0_{ij} - E_{ij,c}) \theta_{ij,t,c} \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^1 \qquad (27w)$$

$$\left| \frac{\psi^0_{ij,t,c,s}}{X_{ij} \rho_s} \right| \le (N^0_{ij} - E_{ij,c}) \overline{P}_{ij,c} z_{ij,t,s} \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^1 \qquad (27x)$$

$$\left| f^0_{ij,t,c} - \frac{\psi^0_{ij,t,c,s}}{X_{ij} \rho_s} \right| \le (N^0_{ij} - E_{ij,c}) \overline{P}_{ij,c} \; (1 - z_{ij,t,s}) \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^1 \qquad (27y)$$

$$\left| f^0_{ij,t,c} \right| \le N^0_{ij} \overline{P}_{ij,c} \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^0 \cup C^2 \cup C^3 \qquad (27z)$$

$$\left| f^0_{ij,t,c} \right| \le (N^0_{ij} - E_{ij,c}) \overline{P}_{ij,c} \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^1 \qquad (27aa)$$

$$\left| f^{t,c}_{ij,y,t,c} X_{ij} - \sum_{s \in S} \psi_{ij,y,t,s,c} - \theta_{ij,t,c} \right| \le M (1 - x_{ij,y,t,c}) \qquad (27bb)$$
$$\forall ij \in \Omega, \forall y \in Y, \forall t \in T, \forall c \in C^0 \cup C^1 \cup \left(C^2 \mid y > 1 \right) \cup C^3$$

$$\left| \frac{\psi_{ij,y,t,s,c}}{X_{ij} \rho_s} \right| \le \overline{P}_{ij,c} u_{ij,y,t,s} \qquad \forall ij \in \Omega, \forall t \in T, \forall s \in S, \forall c \in C^0 \cup C^1 \cup \left(C^2 \mid y > 1 \right) \qquad (27cc)$$

$$\left| \frac{\psi_{ij,y,t,s,c}}{X_{ij} \rho_s} - f_{ij,y,t,c} \right| \le \overline{P}_{ij,c} (1 - u_{ij,y,t,s}) \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T, \forall c \in C^0 \cup C^1 \cup \left(C^2 \mid y > 1 \right) \qquad (27dd)$$

$$\left| f_{ij,y,t,c} X_{ij} - \sum_{s \in S} \psi_{ij,y,t,s,c,} - \theta_{ij,t,c} \right| \le M \left(1 - x_{ij,y,t,c} (1\text{-E}_{ij,c}) \right) \qquad (27ee)$$
$$\forall ij \in \Omega, \forall y \in Y, \forall t \in T, \forall c \in C^2 \mid y = 1$$

$$\left|\frac{\psi_{ij,y,t,s,c}}{X_{ij}\rho_s}\right| \le \bar{P}_{ij,c} u_{ij,t,y,s}(1\text{-E}_{ij,c}) \qquad \forall ij \in \Omega, \forall t \in T, \forall s \in S, \forall c \in (C^2 \mid y=1) \cup C^3 \tag{27ff}$$

$$\left|\frac{\psi_{ij,y,t,s,c}}{X_{ij}\rho_s} - f_{ij,y,t,c}\right| \le \bar{P}_{ij,c}\left(1 - u_{ij,y,t,s}(1\text{-E}_{ij,t})\right) \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T, \forall c \in (C^2 \mid y=1) \cup C^3 \tag{27gg}$$

$$\left|f_{ij,t,y,c}\right| \le x_{ij,t,y}\bar{P}_{ij,c} \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T, \forall c \in C^0 \cup C^1 \cup \left(C^2 \mid y>1\right) \cup C^3 \tag{27hh}$$

$$\left|f_{ij,t,y,c}\right| \le x_{ij,y,t}(1 - E_{ij,c})\bar{P}_{ij,c} \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T, \forall c \in C^2 \mid y=1 \tag{27ii}$$

$$\left|\theta_{ij,t,c}\right| \le \bar{\theta} \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in \left((C^0 \cup C^2 \cup C^3) \mid N_{ij}^0 \ge 1\right) \cup \left(C^1 \mid N_{ij}^0 \ge 2\right) \tag{27jj}$$

$$\left|\theta_{ij,t,c}\right| \le \bar{\theta} + ME_{ij,c}^0 \qquad \forall ij \in \Omega, \forall t \in T, \forall c \in C^1 \mid N^0{}_{ij} = 1 \tag{27kk}$$

$$\left|\theta_{ij,t,c}\right| \le \bar{\theta} + M(1 - x_{ij,y,t}) \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T, \forall c \in \left(C^0 \cup C^1 \cup C^3 \cup (C^2 \mid y \ge 2)\right) \mid N_{ij}^0 = 0 \tag{27ll}$$

$$\left|\theta_{ij,t,c}\right| \le \bar{\theta} + M(1 - x_{ij,y,t}(1 - E_{ij,c})) \qquad \forall ij \in \Omega, \forall y \in Y, \forall t \in T, \forall c \in (C^2 \mid y=1) \mid N_{km}^0 = 0 \tag{27mm}$$

4.3.2.2.1 Objective function

The objective function is comprised of the cost of transmission lines and FSCs of all stages. The first and third terms of (27a) are the investment cost of lines and FSCs in the first planning stage. For planning stage 2 and beyond, the second and fourth terms of objective function calculate the investment cost of lines and FSCs.

4.3.2.2.2 Contingency independent constraints

1) Definition of Variables:

Constraint (27b) defines the maximum and minimum limits of generation. Since generators cannot respond to any contingency immediately, it is considered a contingency independent variable (KAZEROONI; MUTALE, 2010). This is a very conservative assumption since the system may have a kind of spinning reserve that can be accessible immediately. Constraint (27c) defines the investment variables for FSCs ($z_{ij,\mathrm{t},s}$, $u_{ij,y,t,s}$) and transmission lines ($x_{ij,y,t}$) as binary variables.

2) FSC installation constraints:

Constraints (27d) and (27e) enforces the problem to install only one type of the FSCs with a predefined compensation level in existing and candidate lines, respectively. Constraints (27f) and (27g) state that the maximum compensation level for any existing or candidate lines cannot exceed a predefined maximum value. The maximum compensation level for the short lines (less than 200 km) is set to zero. Constraints (27h) and (27i) ensure installation of FSC only in existing and candidate lines. Note that $z_{ij,t,s}$ or $u_{ij,y,t,s}$ cannot be greater than zero unless N_{ij}^{0} or $x_{ij,y,t}$ are greater than zero. These restrictions avoid exploring a solution that allocates FSC in corridors without transmission lines. Constraints (27j) and (27k) are necessary for load balancing in the parallel lines and they state that the same FSCs are installed in all similar lines of a corridor. These constraints become effective if a candidate line is installed, i.e. $x_{ij,y,t} = 1$. Note that (27j) applies for the corridor with existing lines, i.e. $N_{ij}^{0} \geq 1$ and it prevents installation of an FSC in a candidate line without installing FSC in similar existing lines. It also states that if the problem requires installation of an FSC in an existing line, it is not possible to install a similar line without installing an FSC in that line. Also note that constraint (27k) installs similar FSCs in similar candidate lines; in other words, this equation states that if an FSC installed in $(y\text{-}1)^{th}$ line and if the problem requires installation in the y^{th} line, it must also install the same FSC in this line. Constraints (27l) and (27m) guarantee that an FSC installed in a stage must be present in later stages.

3) Candidate lines installation constraints:

The maximum number of lines in a corridor is provided by (27n). Constraint (27o) enforces candidate lines to be installed in sequence. Constraint (27p) guarantees that a line installed in one stage must be present in future stages.

4.3.2.2.3 Contingency dependent constraints

When an outage occurs in a line or an FSC, its power flow or compensation becomes zero, respectively, but it changes the whole system's contingency dependent variables and constraints. Therefore, different operation variables are considered for each contingency and the constraints are repeated for each contingency using these variables. Parameter $E_{ij,c}$ is used to realize the effect of an outage on the model. Note that in this section, the expression of each system constraint should be explored in each system condition, i.e., (C^0, C^1, C^2 and C^3), where only C^0 has a unique member and C^1, C^2 and C^3 may have several members depending on the number of existing lines, candidate lines or FSCs anticipated for outage.

1) Power equilibrium in each system bus:

Constraint (27q) states that regardless of any condition, the power flow equilibrium must be observed at every system bus.

2) Power flow expression in existing lines:

Constraints (27r)-(27y) are used to express the power flow in existing lines in different system conditions. These constraints are explained as follows. Using Figure 12, the power flow through existing lines is given by (28).

$$f^0_{ij,t,c} = \frac{\theta_{ij,t,c}}{(X_{ij} - \sum_{s\in S} X_{ij}\rho_s z_{ij,ts}) / N^0_{ij}} \tag{28}$$

where the denominator is the total impedance of existing lines. Moving the impedance to the left, equation (29) is obtained.

$$f^0_{ij,t,c} X_{ij} - \sum_{s\in S} X_{ij}\rho_s f^0_{ij,t,c} z_{ij,t,s} = N^0_{ij}\theta_{ij,t,c} \tag{29}$$

Equation (29) is nonlinear since there is a product of a binary and a continuous variable on left side. This equation can be linearized by introducing a new continuous variable $\psi^0_{ij,t,c,s} = X_{ij}\rho_s f^0_{ij,t,c,} z_{ij,t,s}$. Note that if $z_{ij,t,s} = 0$, then $\psi^0_{ij,t,c,s} = 0$; otherwise, if $z_{ij,\mathrm{t},s} = 1$, then $\psi^0_{ij,t,c,s} = X_{ij}\rho_s f^0_{ij,t,c}$. Therefore, equation (29) and the expression of $\psi^0_{ij,t,c,s}$ can be considered in linear form in constraints (27r), (27s) and (27t). These three constraints are applied to express power flow when the system is in normal operation (C^0) or contingency in candidate lines (C^2). Constraint (27r) is also correct for contingency in FSCs (C^3); however, the expression of $\psi_{ij,t,c,s}$ in this contingency case is defined in (27u), (27v) using $E_{ij,c}$. Note that for all corridors ($E_{ij,c} = 0$), except the corridor that experiences the FSC outage ($E_{ij,c} = 1$). This parameter also helps in obtaining linearized power flow in cases when an outage occurs in existing lines (C^1). This time $E_{ij,c}$ is set to 1 for the corridor experiencing an outage in the existing line and 0 for other corridors. In this case, the power flow is calculated removing one of the existing lines, which can be expressed mathematically by constraints (27w), (27x) and (27y) .

3) Power flow limit in existing lines:

When the system is in normal condition or contingency (in a candidate line or FSC), the maximum value of power flow in existing lines is defined by (27z). When the system experiences contingency in an existing line, the power flow is defined by (27aa).

4) Power flow expression in candidate line:

According to Figure 12, the active power flow in the y^{th} line of ij^{th} corridor and c^{th} contingency is given by:

$$f_{ij,y,t,c} X_{ij} - \sum_{s \in S} X_{ij} \rho_s f_{jj,y,t,c} u_{ij,y,t,s} = x_{ij,y,t} \theta_{ij,c} \tag{30}$$

This constraint can be linearized by using a new continuous variable, $\psi_{ij,y,t,c,s} = X_{ij} \rho_s f_{ij,y,t,c} u_{ij,\mathrm{t},s}$, and also by using the big-M technique. The linear power flow constraints in candidate lines for normal condition and contingency in existing lines are given by constraints (27bb), (27cc), (27dd). These power flow expressions are also valid for all but the first candidate lines of the contingency in candidate lines, i.e., $\left(C^2 \mid y > 1\right)$. Note that constraint (27bb) is also correct for contingencies in FSC, since $\psi_{ij,y,t,c,s}$ can be set to zero by (27ff) and (27gg) for the FSCs considered for bypass. Finally, the power flow expression in the first candidate line for contingency in a candidate line is expressed in (27ee), (27ff) and (27gg).

5) Power flow limit in candidate line:

The power flow limit for candidate lines in different system conditions are provided in (27hh) and (27ii).

6) Voltage angle constraints:

Constraints (27jj)-(27mm) are considered to limit voltage angle difference between buses with transmission lines, existing or candidate, in different system conditions. This is a practical decision to avoid dynamic instability or other security limits under normal operating conditions (ALGUACIL et al., 2003). Three cases are considered:

- $N_{ij}^0 \geq 2$: regardless of any system conditions, the angle becomes limited as stated in constraint (27jj).

- $N_{ij}^{0} = 1$: constraint (27jj) is applied again for normal condition (C^{0}), contingency in candidate line (C^{2}) and contingency in FSC (C^{3}), since they do not affect existing lines. As indicated in (27kk), when outage occurs in an existing line (C^{1}), the phase angle becomes free for the outage line ($E_{ij,c} = 1$) but will be limited for other corridors ($E_{ij,c} = 0$).

- $N_{ij}^{0} = 0$: the candidate lines define the limit of the phase angle difference. When the system is in condition C^{0}, C^{1} and C^{3}, constraint (27ll) is applied. That means that for $x_{ij,y,t} = 1$ the phase angle becomes limited, otherwise it is free. This constraint is also applied for contingency case C^{2}, but only for the second line and beyond. For the first candidate line, constraint (27mm) comes into play, meaning that in contingency case C^{2} it will not affect the phase angle difference of the corridor experiencing line outage ($E_{ij,c} = 1$), but it will limit it if a line is installed ($x_{ij,y,t} = 1$) in other corridors ($E_{ij,c} = 0$).

TESTS AND RESULTS

In this chapter, we carry out three types of studies, the first is related to the transportation model of transmission expansion planning (TP) (subsection 2.2.5), the second is related to the disjunctive model (DM) (subsection 2.2.3.2) and the reduced disjunctive model (MRDM) (subsection 3.2) and, finally, the third study is related to allocation of fixed series compensation (FSC) in the TEP problem (subsection 4.3.2).

The purpose of studying the TP model is only to show the efficiency of using GRASP-CP for domain reduction of the transmission expansion planning problem. The results of this study are published in (RAHMANI et al., 2012), which are the best for the transportation model according to our knowledge. The results are provided for both transmissions planning with generation and without generation planning. Tests on real systems have been analyzed to show the efficiency of the domain reduction using GRASP-CP. As discussed in section 3.5, there are two important parameters, α and i_{max} , which define the search strategy of the GRASP-CP. The first defines the greediness or randomness of the search and the second defines the number of solutions that must be built to obtain the reduced search space at a later stage. As noted above, these parameters are empirical and in TEP problems, the size of the problem is very important to define these parameters. For small size problems, small numbers of iterations and high value of α may lead us to an optimum solution while in large scale problems, a large number of iterations and a low value of α are selected. In this thesis, different numbers for these parameters are selected in order provide a deeper insight into the method. It should be noted that the GRASP-CP proposed in section 3.5 initially used a hybrid linear model to reduce the search space of the DC model. However, the algorithm can be simply adjusted for transportation model.

In the second study, the reduced disjunctive model is used to solve both the static and the multistage model of transmission expansion planning problem. For comparison purposes the common disjunctive model (DM) is also used to obtain the optimum solutions of the problem. In this study, the three strategies proposed in chapter 3 are used to reduce the search space of the problems. The best results for the DC model are also obtained in these studies for large scale systems (RAHMANI et al., 2013).

In the third study, the allocation of fixed series compensation in the TEP problem, described in chapter 4, is discussed. It is shown that allocation of FSCs can significantly reduce investment in transmission lines through efficient dispatching of the power flow. The results for real and test systems in the multistage TEP with and without FSC and also with and without security constraints are analyzed.

The CPLEX solver 12.4 (IBM ILOG CPLEX, 2012) is used in AMPL (FOURER et al., 2002) modeling language to solve the various models of the TEP problem. The CPLEX is able to utilize all the processors in parallel for solving the problems.

5.1 TRANSPORTATION MODEL STUDIES

The transportation model is presented in section 2.2.5 and is the easiest model in transmission expansion planning studies since it does not contain Kirchhoff's second law, where the voltage angle and consequently the power flow definition by voltage angles are ignored in the formulation. Although the model is easy with respect to other existing models for the transmission expansion planning problem, the optimum solution of the large scale problem is still unknown and the branch and bound fails to obtain a solution for these systems in the full search space of the problem.

In terms of solution methodology, two types of tests have been carried out:

1) The CPLEX branch and bound solver is directly used to solve the problem in the full search space of the problem.

2) The GRASP-CP is used first to reduce the search space of the problem and then the CPLEX branch and bound solver is applied on the reduced search space to obtain the solution.

In terms of the system topology, two different types of planning are carried out:

1) Planning for the base case topology, meaning that some lines are already installed and the system needs to be reinforced in order to supply growing power demand.

2) Planning without the base case topology in which no lines are considered in the system. This type of planning is much more complicated than the first one since many transmission lines need to be installed in the optimum solution.

The studies are also categorized as planning with generation rescheduling and planning without generation rescheduling. In the former case, it is assumed that generator levels can change at a predefined level while in the later, which is much more realistic, variation of generation is not allowed and it is assumed that they are already provided by the power system market.

5.1.1 Southern Brazilian System

This system has 46 buses, 79 circuits and 6880 MW of demand and has been studied in many references including (BINATO et al., 2001a; PEREIRA; GRANVILLE, 1985; RAHMANI et al., 2010; ROMERO et al., 2002). The basic topology of this system and the system data are provided in Appendix I. In buses 28 and 31, new generation units are considered and must be connected to the network. The system has 62 existing transmission lines and 17 new corridors are considered for line installation. The maximum number of transmission lines allowed for installation in each corridor is 3. Therefore, the number of candidate transmission lines for installation is 237 (3×(62+17). The available data allows study of planning with and without generation rescheduling and also with and without base topology. Three tests are performed for each planning to evaluate the behavior of the algorithm.

5.1.1.1 Planning with generation rescheduling

Table 4 shows the results of the tests. Tests 1-3 are provided for planning with base topology and tests 4-6 for planning without base topology. In tests 1 and 4, the branch and bound algorithm is applied on the full space of the problems to obtain the optimum solution while in tests 2, 3, 5 and 6, with different parameters, GRASP-CP along with branch and bound is applied to solve the problem. The number of LP solved (No. LP), the summation of the upper bound of integer variables (SUBINV), which are the total number of candidate transmission lines, the processing time, and the investment costs are given for all tests. In addition, the degree of randomness or greediness (α), the percentage reduction in search space (RSP) and the number of GRASP-CP iteration (GCI) are also provided for tests 2, 3, 5 and 6. In all the tests, the gap between current solutions with respect to the optimum solution is also provided. In test 2 and 5, the optimum solution is not achieved since the GRASP-CP is considered to be too greedy ($\alpha = 0.8$). In tests 3 and 6, the optimum solutions are obtained with a fewer number of solving LP than the BB algorithm applied in full space. The summation of upper bound of variables in tests 3 and 6 are decreased by 90% and 56%, respectively, without losing the optimum solution of the problem.

The investment cost on transmission lines in the optimum solution of TEP with base case topology for the southern Brazilian system is US$53,334×$10^3$ and the following 7 transmission lines are installed: $n_{33\text{-}34}$ = 1, $n_{20\text{-}21}$ = 2, $n_{42\text{-}43}$ = 1, $n_{5\text{-}11}$ = 2, $n_{46\text{-}11}$ = 1. The Investment cost for TEP without base case topology is US$402,748×$10^3$ with the installation of 38 transmission lines:

Table 4 - Southern Brazilian system with generation rescheduling

	With Base Topology			Without Base Topology		
	Test-1 BB	Test-2 GRASP-CP&BB	Test-3 GRASP-CP&BB	Test-4 BB	Test-5 GRASP-CP&BB	Test-6 GRASP-CP&BB
α	-----	0.8	0.6	-----	0.8	0.6
GCI[1]	-----	10	10	-----	20	20
No. LP	16	0	11	11298	1558	3491
SUBINV[2]	237	15	23	237	89	102
Time (sec)	0.2	0.02	0.02	2.65	0.61	1.17
RSP[3] (%)	-----	93	90	----	62	56
gap (%)	0	6.43	0	0	0.87	0
Cost (US$×$10^3$)	53,334*	57,005	53,334*	402,748*	406,288	402,748*

[1] GCI: GRASP-CP Iteration [2] SUBINV: Summation of upper bound of integer variables

[3] Reduction in search space * Optimum solution

Source: The author

$n_{5\text{-}8}$ = 1, $n_{4\text{-}5}$ = 2, $n_{2\text{-}5}$ = 2, $n_{12\text{-}14}$ = 2, $n_{13\text{-}20}$ = 1, $n_{19\text{-}21}$ = 1, $n_{14\text{-}22}$ =1, $n_{22\text{-}26}$ = 1,

$n_{20\text{-}23}$ = 1, $n_{23\text{-}24}$ = 1, $n_{26\text{-}27}$ = 1, $n_{24\text{-}34}$ = 1, $n_{33\text{-}34}$ =1, $n_{27\text{-}36}$ = 1, $n_{34\text{-}35}$ = 1, $n_{37\text{-}40}$ = 1,

$n_{39\text{-}42}$ = 3, $n_{38\text{-}42}$ = 1, $n_{32\text{-}43}$ =1, $n_{42\text{-}44}$ =1, $n_{44\text{-}45}$ = 1, $n_{46\text{-}16}$ = 1, $n_{20\text{-}21}$ = 3, $n_{42\text{-}43}$ = 3,

$n_{46\text{-}6}$ =1, $n_{25\text{-}32}$ = 1, $n_{31\text{-}32}$ = 1, $n_{28\text{-}31}$ = 1, $n_{24\text{-}25}$ = 1, $n_{5\text{-}6}$ = 2.

5.1.1.2 Planning without generation rescheduling

Several tests similar to planning with generation rescheduling have been implemented for both planning with and without base topology. Table 5 shows the results of the tests in which the domain of search space, the processing time and the number of iterations in tests 8, 9, 11 and 12 are reduced significantly. In all tests, except test-8, which is too greedy, the optimum solutions are obtained with little effort, something that can be confirmed by the processing time and the number of LP solved for each test. The Investment cost for TEP without base case topology is US$53,334×$10^3$ with installation of 13 transmission lines: $n_{14-22} = 1$, $n_{20-21} = 2$, $n_{42-43} = 2$, $n_{5-11} = 2$, $n_{25-32} = 1$, $n_{31-32} = 1$, $n_{28-31} = 1$, $n_{46-11} = 1$, $n_{24-25} = 2$.

Table 5 - Southern Brazilian system without generation rescheduling

	With Base Topology			**Without Base Topology**		
	Test-7 BB	Test-8 GRASP-CP&BB	Test-9 GRASP-CP&BB	Test-10 BB	Test-11 GRASP-CP&BB	Test-12 GRASP-CP&BB
α	-----	0.8	0.6	-----	0.8	0.6
GCI[1]	-----	50	50	-----	50	50
No. LP	455	404	155	96466	7052	15026
SUBINV[2]	237	41	51	237	96	113
Time (sec)	0.33	0.05	0.06	9.17	2.13	5.56
RSP[3] (%)	-----	82	78	-----	59	52
gap (%)	0	2.8	0	0	0	0
Cost (US$×$10^3$)	127,272 *	130,943	127,272*	473,246 *	473,246*	473,246*

[1] GCI: GRASP-CP Iteration **[2] SUBINV: Summation of upper bound of integer variables**
[3] Reduction in search space *** Optimum solution**

Source: The author

The solution for this system without base case topology has an investment cost of US$473,246×$10^3$ with installations of 49 transmission lines:

$n_{5-8} = 1$, $n_{4-5} = 2$, $n_{2-5} = 2$, $n_{12-14} = 2$, $n_{13-20} = 1$, $n_{19-21} = 1$, $n_{16-17} = 1$, $n_{17-19} = 1$, $n_{14-26} = 1$,

$n_{14-22} = 1$, $n_{22-26} = 1$, $n_{20-23} = 1$, $n_{23-24} = 1$, $n_{26-27} = 1$, $n_{24-34} = 1$, $n_{24-33} = 1$, $n_{27-36} = 1$, $n_{27-38} = 1$,

$n_{34-35} = 2$, $n_{35-38} = 1$, $n_{37-39} = 1$, $n_{37-40} = 1$, $n_{39-42} = 1$, $n_{38-42} = 1$, $n_{42-44} = 1$, $n_{44-45} = 1$, $n_{46-16} = 1$,

$n_{20\text{-}21}$= 3, $n_{42\text{-}43}$ = 3, $n_{14\text{-}15}$ = 1, $n_{46\text{-}6}$ = 1, $n_{19\text{-}25}$=1, $n_{31\text{-}32}$ = 1, $n_{28\text{-}31}$ = 1, $n_{31\text{-}41}$ = 1, $n_{41\text{-}43}$ = 1, $n_{15\text{-}16}$ = 1, $n_{24\text{-}25}$ = 2, $n_{5\text{-}6}$ = 2.

5.1.2 North-Northeast Brazilian System

The North-Northeast Brazilian System is used as the second case study. This system consists of 87 buses and 183 circuits. The system data is provided in Appendix B. V. This system represents a benchmark in the transmission planning problem due to its high complexity and the unknown global optimal solution. There are two levels of demand, one considered for 2002 (P1) with a level of 20316 MW and the other for 2008 (P2) with a level of 29748 MW.

In the following subsections, several tests of domain reduction, for both plans P1 and P2, with different parameters have been implemented. The tests are carried out with fixed generator levels, i.e. without generation rescheduling but with and without base topology.

5.1.2.1 North-Northeast Brazilian system plan (2002)

In this test it is considered that the system already has a base topology and the system must be reinforced to meet the growing demand.

a) Test with base case topology:

The optimum solution for this plan has an investment cost of US\$1,194,561×10^3; this solution has been reported as the optimum solution in (ROMERO et al., 2002). We also obtain this solution using both branch and bound in the full space of the problem as well as in the reduced space of the problem. 54 transmission lines are installed in the optimum solution as follows:

$n_{2\text{-}60}$ = 2, $n_{5\text{-}58}$ = 2 , $n_{5\text{-}60}$ = 2, $n_{5\text{-}68}$ = 1, $n_{8\text{-}17}$ = 1, $n_{8\text{-}62}$ = 2, $n_{9\text{-}10}$ = 1, $n_{10\text{-}11}$ = 1, $n_{11\text{-}17}$ = 1, $n_{13\text{-}15}$ = 2, $n_{14\text{-}59}$ = 1, $n_{15\text{-}16}$ = 2, $n_{16\text{-}44}$ = 3, $n_{17\text{-}18}$ = 2, $n_{18\text{-}50}$ = 6, $n_{20\text{-}21}$ = 1, $n_{20\text{-}38}$ = 1, $n_{24\text{-}43}$ = 1, $n_{25\text{-}55}$ = 1, $n_{30\text{-}63}$ = 1, $n_{35\text{-}51}$ = 1, $n_{40\text{-}45}$ = 1, $n_{41\text{-}64}$ = 3, $n_{42\text{-}44}$ = 2, $n_{42\text{-}85}$ = 1, $n_{43\text{-}55}$ = 1, $n_{43\text{-}58}$ = 1, $n_{48\text{-}49}$ = 3, $n_{54\text{-}58}$ = 1, $n_{54\text{-}63}$ = 1, $n_{62\text{-}67}$ = 2, $n_{63\text{-}64}$ = 1, $n_{67\text{-}69}$ = 1, $n_{69\text{-}87}$ = 1.

Table 6 shows the results of the tests for this system. Three tests have been carried out. In test-13, the CPLEX branch and bound solver is applied to solve the problem in full space while in tests 14 and 15, the GRASP-CP is applied to reduce the search space of the problem prior to applying branch and bound. In test14, the optimum solution is not obtained since the

GRASP-CP is too greedy. In tests 14 and 15, we observe a domain reduction of over 85%, resulting in less processing time and a lesser number of LP while obtaining the optimum solution.

Table 6 - North-Northeast Brazilian system without generation rescheduling and with base case topology for Plan 2002

	With Base Case Topology		
	Test-13 BB	Test-14 GRASP-CP&BB	Test-15 GRASP-CP&BB
α	-----	0.6	0.1
GCI[1]	-----	100	100
No. LP	179156	28619	74173
SUBINV[2]	2700	203	360
Time (sec)	23.57	8.46	14.06
RSP[3] (%)	---	99.14	86.66
gap (%)	0	3.58	0
Cost (US$×10^3)	1,194,561*	1,238,944	1,194,561*

[1] GCI: GRASP-CP Iteration

[2] SUBINV: Summation of upper bound of integer variables

[3]Reduction in search space *** Optimum solution**

Source: The author

5.1.2.2 Test without base case topology

No study has been done on for obtaining the solution of the North-Northeast Brazilian system for the transportation model without existing topology. Table 7 shows the results for the transportation model. Similar to the previous subsection, three tests were carried out. In test-16, Table 7, branch and bound was applied in the full space of the problem but after solving millions of LP and several days of running (more than 365000 sec), the optimum solution was not obtained and the process was stopped due to the lack of memory. However, a very good solution with an optimality gap of less than 1% was obtained. In test 17 and 18, the GRASP-CP, with a different parameter, was applied to reduce the search space of the problem prior to using BB solver. The obtained solutions have a very small (<1%) gap with respect to the best solution obtained in test-16. Although the solution obtained in tests 17 and 18 are not better to that obtained in 16, a significant reduction in search space observed in them, resulting in a lower number of LP and much less computation hours.

Table 7 - North-Northeast Brazilian system without generation rescheduling and without base case topology for Plan 2002.

	Without Base Topology		
	Test-16 BB	Test-17 GRASP-CP&BB	Test-18 GRASP-CP&BB
α	-----	0.6	0.1
GCI[1]	-----	100	100
No. LP	1.51×10^{9}	7.61×10^{7}	1.2×10^{8}
SUBINV[2]	2700	331	432
Time (sec)	365400	16694	31585
RSP[3] (%)	---	87.74	84.00
gap (%)	0.73	0.788	0.783
Cost (US$×10³)	2,344,023♦	2,356,727	2,355,601

[1] GCI:GRASP-CP Iteration

[2] SUBINV: Summation of upper bound of integer variables

[3] Reduction in search space *** Optimum solution** **♦ Best solution**

Source: The author

The best solution obtained for this system needs 125 transmission lines and the investment cost is about US$2,344,023×10³ with the following installation:

$n_{1-2}=2$, $n_{2-4}=1$, $n_{2-60}=2$, $n_{4-5}=1$, $n_{4-69}=1$, $n_{5-56}=1$, $n_{5-58}=3$, $n_{5-60}=2$,$n_{5-68}=1$, $n_{7-8}=1$, $n_{7-62}=1$, $n_{8-17}=2$, $n_{8-53}=1$, $n_{8-62}=1$, $n_{9-10}=1$, $n_{10-11}=2$,$n_{11-15}=2$, $n_{11-17}=1$, $n_{12-17}=3$, $n_{12-35}=2$, $n_{13-15}=3$, $n_{13-45}=1$, $n_{14-59}=1$, $n_{15-16}=4$, $n_{15-46}=1$, $n_{16-44}=7$, $n_{16-61}=1$, $n_{17-18}=6$, $n_{18-50}=11$, $n_{19-20}=1$, $n_{20-21}=2$, $n_{20-21}=1$, $n_{20-56}=2$, $n_{22-23}=1$, $n_{22-37}=1$, $n_{22-58}=1$, $n_{24-43}=1$, $n_{25-26}=1$, $n_{25-55}=3$, $n_{26-29}=1$, $n_{27-28}=2$, $n_{27-53}=1$, $n_{30-3}=1$, $n_{30-63}=1$, $n_{34-39}=1$, $n_{34-41}=1$, $n_{35-51}=3$, $n_{36-39}=1$, $n_{36-46}=2$, $n_{39-42}=1$, $n_{40-45}=2$, $n_{40-46}=1$, $n_{41-64}=3$, $n_{42-44}=1$, $n_{42-85}=1$, $n_{43-55}=2$, $n_{43-58}=2$, $n_{48-49}=1$, $n_{48-50}=3$, $n_{49-50}=4$, $n_{51-52}=1$, $n_{52-59}=1$, $n_{54-58}=1$, $n_{54-63}=1$, $n_{61-85}=2$, $n_{62-67}=2$, $n_{63-64}=1$, $n_{67-69}=1$.

5.1.2.3 North-Northeast Brazilian System Plan 2 (2008)

- Tests with base topology

The optimum solution of this system is not given in literature, due to the high complexity of the problem. In test 19, Table 8, the optimum solution of this system is obtained with the CPLEX branch and bound solver in the full space of the problem. In test-20, GRASP-CP with $\alpha = 0.6$ is used to reduce the search space of the problem. The results have an optimality gap of about 1.14%, therefore in test-21, we make the algorithm much more random using $\alpha = 0.1$ to obtain a better result. In test-21, a solution with an optimality gap of about 0.74% is obtained which is a very good result. The reduction in search space is significant (over 80%) and the time for achieving this solution is much less than the branch and bound in full space.

Table 8 - North-Northeast Brazilian system without generation rescheduling, with base case topology for Plan 2008

	With base case topology		
	Test-19 BB	Test-20 GRASP CP&BB	Test-21 GRASP-CP&BB
α	-----	0.6	0.1
GCI[1]	-----	100	100
No. LP	1.42×10^{8}	1.32×10^{6}	3.26×10^{6}
SUBINV[2]	2700	305	518
Time (sec)	1299	193	419
RSP[3] (%)	0	88.73	80.81
gap (%)	0	1.14	0.74
Cost (US$×10[3])	2,370,680*	2,398,025	2,388,359

Source: The author

The optimum solution has an investment cost of about US\$ $2{,}370{,}680\times10^3$ with an installation of 95 lines in the optimum solution as follows:

$n_{1-2}=1$, $n_{2-60}=1$, $n_{4-5}=2$, $n_{4-6}=1$, $n_{4-68}=1$, $n_{4-81}=3$, $n_{5-58}=3$, $n_{5-60}=1$, $n_{13-15}=4$, $n_{14-45}=1$,

$n_{15-16}=4$, $n_{16-44}=6$, $n_{16-61}=1$, $n_{18-50}=11$, $n_{18-74}=6$, $n_{20-21}=2$, $n_{20-38}=2$, $n_{22-23}=1$, $n_{22-58}=2$,

$n_{24-43}=1$, $n_{25-55}=3$, $n_{26-29}=2$, $n_{29-30}=2$, $n_{39-86}=4$, $n_{40-45}=2$, $n_{41-64}=2$, $n_{42-44}=1$, $n_{43-55}=2$,

$n_{43-58}=2$, $n_{48-49}=2$, $n_{49-50}=3$, $n_{52-59}=1$, $n_{53-86}=1$, $n_{61-64}=1$, $n_{61-85}=2$, $n_{67-68}=1$, $n_{67-69}=1$,

$n_{67-71}=3$, $n_{71-72}=1$, $n_{72-73}=1$, $n_{73-74}=2$, $n_{73-75}=1$, $n_{75-81}=1$.

- Tests without base topology

This system is very complicated; the optimum solution is not obtained and the optimality gap is not satisfying. However, the best solution for this problem is obtained using GRASP-CP and branch and bound. The test with application of branch and bound without using GRASP-CP, Table 9 test-22, fails to obtain the optimum solution due to a lack of memory. The solution obtained after several days of running has an optimality gap of about

1.73%. However, test-23 and test-24 obtain solutions in a few hours with an optimality gap that is very approximate to this value. The reduction in search space is more than 77%.

Table 9 - North-Northeast Brazilian system without generation rescheduling, without base case topology for Plan 2008

	Without Base Topology		
	Test-22 BB	Test-23 GRASP-CP&BB	Test-24 GRASP-CP&BB
α	-----	0.6	0.1
GCI[1]	-----	100	100
No. LP	1.24×10^{9}	1.01×10^{6}	7.10×10^{7}
SUBINV[2]	2700	436	596
Time (sec)	249794	11238	20750
RSP[3] (%)	0	83.85	77.92
gap (%)	1.73	1.77	1.70
Cost (US$×10³)	3,534,832	3,536,290	3,533,929♦

Source: The author

The performance of this method is revealed in test-24 in which the GRASP-CP along with BB finds a better solution than the BB applied in the full space of the problem. The former solution is obtained in about 6 hours while the later one obtained in about 70 hours. The investment cost for this system is US$3,533,929×10^3 and 174 transmission lines are installed. Note that this solution is obtained in test-24 in which the GRASP-CP is applied to reduce the search space of the problem:

$n_{1-2}=3$, $n_{2-60}=2$, $n_{4-6}=2$, $n_{4-60}=1$, $n_{4-68}=1$, $n_{4-81}=3$, $n_{5-56}=1$, $n_{5-58}=3$, $n_{5-60}=2$, $n_{5-68}=2$, $n_{6-70}=2$, $n_{7-53}=1$, $n_{7-62}=1$, $n_{8-9}=1$, $n_{8-62}=1$, $n_{9-10}=2$, $n_{10-11}=2$, $n_{11-12}=1$, $n_{11-15}=1$, $n_{11-17}=2$, $n_{12-15}=2$, $n_{12-17}=3$, $n_{12-35}=2$, $n_{13-15}=3$, $n_{13-45}=1$, $n_{13-59}=1$, $n_{14-45}=1$, $n_{15-16}=5$, $n_{15-46}=2$, $n_{16-44}=10$, $n_{17-1}=5$, $n_{18-50}=16$, $n_{18-74}=3$, $n_{19-20}=1$, $n_{19-22}=1$, $n_{20-21}=1$, $n_{20-66}=2$, $n_{21-57}=2$, $n_{22-23}=1$, $n_{22-37}=1$, $n_{22-58}=2$, $n_{24-43}=1$, $n_{25-55}=4$, $n_{26-27}=2$, $n_{26-2}=2$, $n_{27-28}=1$, $n_{27-53}=2$, $n_{28-35}=1$, $n_{29-30}=2$, $n_{30-31}=1$, $n_{30-63}=1$, $n_{31-34}=1$, $n_{34-39}=1$, $n_{35-51}=2$, $n_{36-39}=1$, $n_{36-46}=3$, $n_{39-86}=1$, $n_{40-45}=3$, $n_{41-64}=3$, $n_{42-44}=3$, $n_{43-55}=3$, $n_{43-58}=3$, $n_{44-46}=2$, $n_{48-50}=3$, $n_{49-50}=7$, $n_{51-52}=1$, $n_{52-59}=2$, $n_{53-70}=1$, $n_{53-86}=1$, $n_{54-63}=1$, $n_{54-70}=1$, $n_{56-57}=1$, $n_{60-66}=1$, $n_{61-85}=3$, $n_{61-86}=1$, $n_{62-67}=2$, $n_{63-64}=1$, $n_{67-69}=1$, $n_{68-69}=1$, $n_{73-74}=1$, $n_{73-75}=1$, $n_{75-81}=1$.

5.2 STUDIES ON THE DC MODEL WITH DISJUNCTIVE REPRESENTATION

The common disjunctive model (DM) represented in section 2.2.3.2 and the reduced disjunctive model (RDM) proposed in section 3.2 are used to carry out the tests. The results of RDM are compared with the DM as well as with other available results from literature to demonstrate the performance of the RDM model. For static planning, the Garver 6 bus system (shown in Figure 8), the South Brazilian 46 bus system (shown in Figure 19), the Colombian 93 bus (shown in Figure 20) and the 87 bus Brazilian North-Northeast system (shown in Figure 21) are studied.

In multistage planning, the Colombian and Brazilian North-Northeast systems are examined. The data for these systems can be found in (ESCOBAR, 2004; GALLEGO et al., 2000; ROMERO et al., 2002) and also in Appendix II.

For both static and multistage planning, the optimum solution for all systems, except the Brazilian North-Northeast system which is a highly complicated problem, are obtained. The Brazilian North-Northeast is one of the most complicated systems in transmission expansion planning since it has many candidate lines and needs many line installations for its solution. The optimum solution of this system for the DC model or equivalent DM/RDM is still unknown and the best solutions for static and multistage planning are reported in (ESCOBAR et al., 2004; GALLEGO et al., 2000). For many years no report has been published regarding how to improve these solutions. In this thesis, a significant improvement to the solution for both static and multistage planning is reported.

In section 5.1, we have already shown the excellent performance of the GRASP construction phase (GRASP-CP) for various test systems. In the following section, we primarily focus on a new representation of the disjunctive model (RDM) and also on fence constraints. The GRASP-CP is only applied to the Brazilian North-Northeast since it is not possible to explore the entire search space in polynomial time.

5.2.1 Static Planning

The Garver, South-Brazilian, Colombian and Brazilian North-Northeast systems each with, respectively, 15, 79, 155 and 183 candidate lines and 5, 3, 5 and 12 maximum transmission lines in each corridor are studied in this section. In fact, the maximum number of transmission lines in the Brazilian North-Northeast system is not provided but it is assumed to be large. However, we consider that a maximum of 12 lines can be added in each corridor.

Four tests have been carried on case studies: test-1, disjunctive model (DM); test-2, reduced disjunctive model (RDM); test-3, RDM along with fence constraints in (14a), (15f), (16a) and (17b); and test-4: RDM along with fence constraints in (14a), (15f), (16a) and (17b) as well as the GRASP-CP domain reduction. Test-4 is applied only to the Brazilian North-Northeastern system since in tests 1 to 3, the branch and bound solver is unable to converge to a high-quality solution within a reasonable amount of time due to the enormous size of the system. Table 10 shows the results of tests 1-3 for the Garver, the South-Brazilian and the Colombian networks. The number of binary variables, the processing time for each test, the number of fence constraints in test-3 and the optimum solution of case studies are also provided in this table. Considering the data given in Table 10, the number of binary variables in the Garver, the South-Brazilian and the Colombian systems are decreased by 40%, 33%, and 40% respectively in tests 2 and 3. When the binary numeral system (BNS) is used in the DM, the processing time decreases significantly from test-1 to test-2. However, the reduction in time from test-2 to test-3 depends on the number of fence constraints added to the problem. For example, since it is not possible to add many efficient fence constraints in the Colombian system due to the low power deficit in nodes or supernodes, the improvement in processing time from test-2 to test-3 is trivial, but the reduction in processing time for the Garver system is significant since it is possible to add many strong fence constraints. The best results are obtained in test-3 in terms of computational time.

The installed lines for Garver and South Brazilian networks are provided in (ROMERO et al., 2007). For the Colombian system, the installed lines in the optimum solution are provided in (VERMA et al., 2010) and also as follows*:*

$n_{43\text{-}88} = 2$, $n_{15\text{-}18} = 1$, $n_{30\text{-}65} = 1$, $n_{30\text{-}72} = 1$, $n_{55\text{-}57} = 1$, $n_{55\text{-}84} = 1$, $n_{56\text{-}57} = 1$, $n_{55\text{-}62} = 1$, $n_{27\text{-}29} = 1$, $n_{27\text{-}64} = 1$, $n_{50\text{-}54} = 1$, $n_{62\text{-}73} = 1$, $n_{54\text{-}56} = 1$, $n_{72\text{-}73} = 1$, $n_{19\text{-}82} = 2$, $n_{82\text{-}85} = 1$, $n_{68\text{-}86} = 1$.

In test-4, the Brazilian North-Northeast system is studied. The number of GRASP-CP iterations for configuring a reduced space search as well as proposing an initial incumbent solution is set to 100 and the value for α is considered to be 0.6.

Table 10 - Results of static TEP for Garver, South-Brazilian and Colombian networks

System	Test-1		Test-2		Test-3			Optimal Cost $\times 10^3$ US$
	Bin No.	Time (sec)	Bin No.	Time (sec)	Bin No	Time (sec)	Fence Constraints	
GARV	75	0.09	45	0.06	45	0.03	139	200
SB	237	13.31	158	10.25	158	8.58	23	154,420
COL	775	2161	465	541	465	529	2	562,417

GARV: Garver 6 bus system 15×5 = 75 full candidate lines (DM)
SB: South Brazilian 46 bus system 79×3 = 237 full candidate lines (DM)
COL: Colombian system 155×5 = 775 full candidate lines (DM)

Source: The author

Table 11 shows the results for the Brazilian North-Northeast system for static planning. The number of binary variables decreased from 2196 in the DM to 732 in the RDM and 176 when GRASP-CP is applied, i.e. about 66% in the first and 92% in the second consideration. The obtained solution is better than other known solutions found in the literature.

Table 11 - Results of static TEP for Brazilian North-Northeast

System	Test-4: RDM & Special Constraints & GRASP-CP				
	Binary	Time (hr)	Fence Constraint	Cost $\times 10^3$ US$	Best Known $\times 10^3$ US$
BNN	176	55	411	2,546,417	2,574,745

BNN: Brazilian North-Northeast;
183×12=2196 full candidate lines (DM)
183×4=732 candidate lines in (RDM)

Source: The author

The best known solution for the Brazilian North-Northeast system, reported in (GALLEGO et al., 2000), has a total cost of about US$2,574,745×$10^3$ while the solution obtained in this thesis requires an investment of US$2,546,417×$10^3$, which is US$28,328×10^3 less, and proposes the following additions:

$n_{1-2} = 1$, $n_{2-87} = 1$, $n_{4-5} = 4$, $n_{4-68} = 1$, $n_{4-81} = 3$, $n_{5-56} = 1$,

$n_{5-58} = 3$, $n_{6-37} = 1$, $n_{12-15} = 1$, $n_{13-15} = 4$, $n_{14-45} = 1$, $n_{15-16} = 4$,

$n_{15-46} = 1$, $n_{16-44} = 6$, $n_{16-61} = 2$, $n_{18-50}=11$, $n_{18-74} = 6$, $n_{21-57} = 2$,

$n_{22-23} = 1$, $n_{22-37} = 2$, $n_{25-55} = 4$, $n_{26-54} = 1$, $n_{27-53} = 1$, $n_{29-30} = 1$,

$n_{30-31} = 2$, $n_{30-63} = 2$, $n_{35-51} = 2$, $n_{36-39} = 1$, $n_{36-46} = 3$, $n_{40-45} = 2$,

$n_{41-64} = 2$, $n_{43-55} = 2$, $n_{43-58} = 2$, $n_{48-49} = 1$, $n_{49-50} = 4$, $n_{52-59} = 1$,

$n_{54-58} = 1$, $n_{54-63} = 1$, $n_{56-57} = 1$, $n_{61-64} = 1$, $n_{61-85} = 3$, $n_{67-69} = 2$,

$n_{67-71} = 3$, $n_{69-87} = 1$, $n_{71-72} = 1$, $n_{72-73} = 1$, $n_{73-74} = 2$, $n_{73-75} = 1$,

$n_{75-81} = 1$.

To test the optimality of this solution, we set it as the incumbent solution in test-3, the RDM along with fence constraints and run the program for 15 days, considering the full space of the problem. Although the CPLEX branch and bound could not find a better solution than test-4, during this running time, it demonstrated that the gap of this solution to the optimum solution is about 5%. This highlights the proximity of the proposed solution to the optimum solution and the efficiency of the GRASP-CP domain reduction in MILP problems. The gap of a solution with respect to the optimum solution is obtained by considering that the optimal value of an integer program is bounded on one side by the best integer (incumbent) objective value found so far, and on the other side, by a value deduced from all the relaxed LP nodes of sub-problems solved so far (IBM ILOG CPLEX, 2012).

5.2.2 Multistage Planning

The Colombian system with three-stage planning, namely P_1, P_2 and P_3 and the Brazilian North-Northeast systems with two-stage planning, namely P_1 and P_2, are considered for carrying out studies. The optimum solution for the Colombian system in multistage planning is reported in (VINASCO et al., 2011), with a total investment equal to US\$492,167×10^3. As in static planning, three tests are carried out for this system: one test with DM and two other tests using RDM with and without special constraints. The GRASP-CP phase is not considered in this test since the solution can be found within a reasonable amount of time. Table 12 shows the results of the tests of the Colombian system in which there is a significant improvement in computational time from test-1 (DM) to test-2 (RDM). As in the static model, only a little improvement can be observed from test-2 to test-3 since the number of fence constraints is too low.

The multistage TEP of the Colombian system is not a very challenging problem since only a maximum of 2 lines is installed in some corridors and a total of 19 transmission lines configure the optimal solution. Also, it should be noted that it is not possible to insert more than two fence constraints in this system, which results in the small difference between tests 2 and 3.

Table 12 - Results of MTEP for the Colombian system

System	Test-1		Test-2		Test-3			Optimal Cost $\times 10^3$ US$
	Bin No.	Time (sec)	Bin No.	Time (sec)	Bin No	Time (sec)	Fence Constraint.	
COL	325	9886	1395	3040	1395	3016	2	49,167

COL: Colombian system
155×3×5 = 2325 full candidate lines (DM)
155×3×3 = 1395 Candidate lines (RDM)

Source: The author

In multistage planning for the Brazilian North-Northeast, all the tests have failed to converge to a high-quality solution, once again due to the enormous size of the problem and, as in static planning, the GRASP-CP is employed to reduce the search space. The value of α is set to 0.9, and 20 iterations of GRASP-CP are implemented to reduce the search space. A lower value for α and a higher number of iterations of GRASP-CP lead to an extensive search space and a much higher processing time.

The best topology was found with the actual value of the investment projected to the base year equal to US$2,197.55×$10^6$ which is US$6.73×$10^6$ less than the best reported investment of US$2,204.28×$10^6$ in (ESCOBAR, A.H. et al., 2004). The proposed topology for two stages of planning for the Brazilian North-Northeast is as follows:

Stage P_1: $v_1 = 1.00 \times$ US$1,419.14×$10^6$:

$n_{2\text{-}4} = 1$, $n_{2\text{-}60} = 1$, $n_{4\text{-}5} = 1$, $n_{5\text{-}58} = 2$, $n_{5\text{-}60} = 1$, $n_{12\text{-}15} = 1$,

$n_{13\text{-}15} = 2$, $n_{14\text{-}45} = 1$, $n_{15\text{-}16} = 2$, $n_{16\text{-}44} = 3$, $n_{16\text{-}61} = 1$, $n_{18\text{-}50} = 6$,

$n_{18\text{-}74} = 3,\ n_{20\text{-}21} = 2,\ n_{20\text{-}38} = 1,\ n_{22\text{-}58} = 1,\ n_{24\text{-}43} = 1,\ n_{25\text{-}55} = 2,$

$n_{26\text{-}29} = 2,\ n_{26\text{-}54} = 1,\ n_{27\text{-}53} = 1,\ n_{29\text{-}30} = 1,\ n_{35\text{-}51} = 1,\ n_{36\text{-}39} = 1,$

$n_{36\text{-}46} = 2,\ n_{40\text{-}45} = 2,\ n_{41\text{-}64} = 2,\ n_{43\text{-}55} = 1,\ n_{43\text{-}58} = 1,\ n_{49\text{-}50} = 3,$

$n_{54\text{-}58} = 1,\ n_{61\text{-}64} = 1,\ n_{61\text{-}85} = 2,\ n_{67\text{-}68} = 1,\ n_{67\text{-}69} = 1,\ n_{67\text{-}71} = 3,$

$n_{71\text{-}72} = 1,\ n_{72\text{-}73} = 1,\ n_{73\text{-}74} = 1$

Stage P_2: $v_2 = 0.656 \times$ US\$1,186.61$\times 10^6$:

$n_{1\text{-}2} = 1,\ n_{4\text{-}5} = 2,\ n_{4\text{-}81} = 3,\ n_{5\text{-}38} = 1,\ n_{5\text{-}58} = 2,\ n_{13\text{-}15} = 2,$

$n_{15\text{-}16} = 2,\ n_{15\text{-}46} = 1,\ n_{16\text{-}44} = 3,\ n_{16\text{-}61} = 1,\ n_{18\text{-}50} = 5,\ n_{18\text{-}74} = 3,$

$n_{20\text{-}21} = 1,\ n_{20\text{-}38} = 1,\ n_{20\text{-}66} = 1,\ n_{22\text{-}58} = 1,\ n_{25\text{-}55} = 1,\ n_{26\text{-}29} = 1,$

$n_{26\text{-}54} = 1,\ n_{29\text{-}30} = 2,\ n_{30\text{-}31} = 1,\ n_{35\text{-}51} = 1,\ n_{36\text{-}46} = 1,\ n_{42\text{-}85} = 1,$

$n_{43\text{-}55} = 1,\ n_{43\text{-}58} = 1,\ n_{49\text{-}50} = 2,\ n_{52\text{-}59} = 1,\ n_{61\text{-}85} = 1,\ n_{65\text{-}66} = 1,$

$n_{65\text{-}87} = 1,\ n_{73\text{-}74} = 1,\ n_{73\text{-}75} = 1,\ n_{75\text{-}81} = 1$

As shown in Table 13, in DM, 4392 binary variables are needed. This amount can be decreased in the RDM to 1464 and decreased even further to 237 when GRASP-CP is accompanied by RDM. Other useful data is also provided in this table.

Table 13 - Results of MTEP for the Brazilian North-Northeast

System	Test-4: RDM & Special Constraints & GRASP-CP				
	Binary	Time (hr)	Fence Constraint	Cost $\times 10^6$ US\$	Best Known $\times 10^6$ US\$
BNN	237	70	644	2,198.06	2,204.28

BNN: Brazilian North-Northeast
183×2×12= 4392 Full candidate lines (DM)
183×2×4= 1464 Candidate lines in (RDM)

Source: The author

The number of lines installed for this system is 112. Comparing this number with the one installed in the Colombian system shows the level of complexity of the Brazilian North-Northeast system.

5.3 STUDIES ON TEP WITH FIXED SERIES COMPENSATION ALLOCATION

The IEEE 24 bus system and the Colombian system, each with three planning stages, are studied. For each system, planning is carried out both with and without FSC allocation. With no loss of generality, the following assumptions are considered:

1) Three types of FSCs with different compensation percentages can be installed in a transmission line as follows: ρ_S =20% with C_s =10%, ρ_S =30% with C_s =15% and ρ_S =50% with C_s =25%.
2) Because the line lengths are not provided in the system data, FSC is allocated in lines with a reactance greater than 0.05 pu.
3) The discount factors due to installation delay are equal to 1.0, 0.729 and 0.478 for the first, second and third stages, respectively.

5.3.1 IEEE-24 Bus Test System

This system has 24 buses, 38 existing lines and 41 candidate transmission lines. The original data are provided in (SUBCOMMITTEE, 1979), and the candidate transmission lines are provided in (FANG; HILL, 2003). The system is to be expanded in three stages to future conditions, with the generator and load levels at 3, 3.05 and 3.1 times their original values. The voltage angle difference between buses with transmission lines is considered to be no greater than 20°. Three tests are carried out for this system: 1) multistage TEP without FSC and without N-1 contingency, 2) multistage TEP without FSC but with N-1 contingency in lines, 3) multistage TEP with FSC and with N-1 contingency in lines and FSCs. The contingency list for tests 2 and 3 is given in Table 14, where columns one, two and three are anticipated contingencies in existing lines, candidate lines and FSCs, respectively. In this thesis, the contingency list is selected manually both from candidate lines and existing lines. However, a better index for selecting contingency lists can be obtained using a solution of TEP without contingency. Silva (2012) used the lines with power flow greater than 80% of their maximum capacity as the most probable lines for outage.

Test 1: multistage TEP without FSC and without N-1 contingency: The optimum solution for this test is obtained in 14 seconds and it has an investment cost equal to US$260.06×10^6. The following lines are installed in different stages:

- Stage 1 (US$22×10^6): $n_{6\text{-}10} = 1$, $n_{7\text{-}8} = 2$, $n_{10\text{-}12} = 1$, $n_{12\text{-}13} = 1$, $n_{14\text{-}16}=1$,
- Stage 2 (US$ 0): no line is installed,
- Stage 3 (US$42.06×10^6): $n_{1\text{-}5} = 1$, $n_{3\text{-}24} = 1$, $n_{7\text{-}8} = 1$.

Table 14 - Contingency list in the IEEE-24 bus system

Existing lines	Candidate lines	FSC
1 – 2	6 – 7	3- 9
1 – 3	13 – 14	5-10
1 – 5	14 – 23	7 – 8
2 – 4	16 – 23	9 -11
2 – 6	19 – 23	12-23

Source: The author

Test 2: multistage TEP without FSC but with N-1 contingency in lines: in this test, 10 lines, existing and candidate, are assumed for possible outages (see Table 14). The optimum solution for this test is obtained in 1.3 hours. It has an investment cost of US$359.11×10^6 and the following lines are installed:

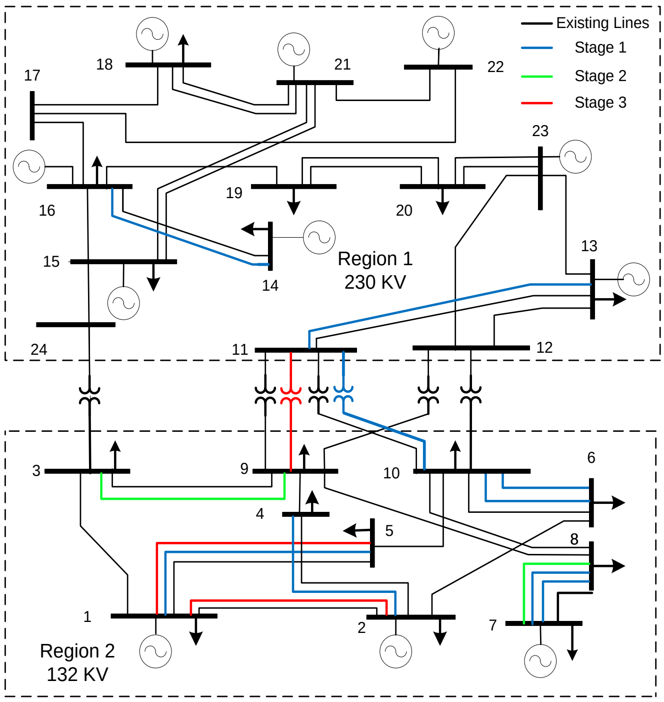
- Stage 1 (US$289×10^6): $n_{1\text{-}5} = 1$, $n_{2\text{-}4} = 1$, $n_{6\text{-}10} = 2$, $n_{7\text{-}8} = 2$, $n_{10\text{-}11} = 1$, $n_{11\text{-}13} = 1$, $n_{14\text{-}16} = 1$,
- Stage 2 (US$34.26×10^6): $n_{3\text{-}9} = 1$, $n_{7\text{-}8} = 1$,
- Stage 3 (US$35.85×10^6): $n_{1\text{-}2} = 1$, $n_{9\text{-}11} = 1$, $n_{1\text{-}5} = 1$.

The installed transmission lines for planning without FSC are depicted in Figure 13, where the lines installed in the first, second and third stages appear in blue, green and red, respectively. 14 lines are installed in this test, while in planning without contingency, 9 lines were installed. 6 lines are common in both types of planning. There are 3 lines that are installed only in planning without contingency while 8 lines just appear in planning with contingency. Of 10 lines considered for outage, only 3 appeared in the solution for TEP without contingency.

Test 3: multistage TEP with FSC and with N-1 contingency in lines and FSCs: in this case, the processing time increases to about 17 hours and the total investment cost decreases to US$311.80×10^6 (US$289×10^6 for lines and US$22.80×10^6 for FSC) to install 9 transmission lines and 7 fixed series compensators as follows:

- Stage 1 (US$289×10^6): lines $n_{1-5} = 1$, $n_{2-4} = 1$, $n_{6-10} = 2$, $n_{7-8} = 2$, $n_{10-11} = 1$, $n_{11-13} = 1$, $n_{14-16} = 1$,
- Stage 2 (US$9.15×10^6): FSC in existing lines (US$6.82×10^6) $z_{3\text{-}9,3}$, $z_{7\text{-}8,1}$ and FSC in new lines (US$2.33×10^6) $u_{7\text{-}8,1}$,
- Stage 3 (US$13.65×10^6): FSC in existing lines $z_{5\text{-}10,2}$, $z_{9\text{-}11,1}$, $z_{12\text{-}23,2}$.

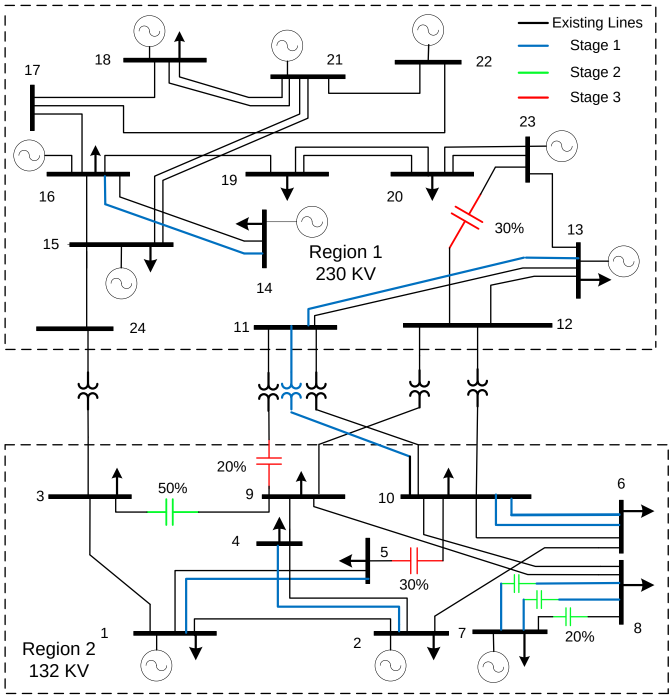
Figure 13 - Installed lines for the IEEE-24 bus system in TEP without FSC and with N-1 contingencies in lines



Source: The author

Figure 14 shows the installed transmission lines and FSCs for this test. It can be observed that 5 transmission lines are removed from the planning without FSC and 7 capacitors are added in various corridors for better use of system transfer capacity. As shown in Figure 13 and Figure 14, there are two voltage levels in the IEEE 24-bus system: the upper part (region 1) with 230 kV and the lower part (region 2) with 132 kV. Region 1 has a generation surplus, whereas region 2 has a generation deficit. Therefore, transmission lines connecting these regions are responsible for transferring power from region 1 to region 2. Table 15 shows the power flow in transmission lines between these regions, for both planning problems, in the third planning stage and for normal condition. It can be observed that the total power flow in tie lines increased by 237 MW from multistage TEP planning without FSC to multistage TEP with FSC, showing better utilization of the system transfer capacity.

Figure 14 - Installed lines and FSCs for the IEEE-24 bus system in TEP with FSC and with N-1 contingency in lines and FSCs



Source: The author

In multistage TEP with FSC, a line installation in corridor 9-11 is avoided by FSC installation in that line and also using the remaining capacity of line 12-9 by installing an FSC in line 12-23. Installing an FSC in line 12-23 adjusts the voltage angle at both its ends and then increases its power flow and, as a result, the power flow through line 12-9 increases. 2 lines have been avoided in corridors 3-9 and 7-8 by installing FSCs in their lines. However, the excellent performance of multistage TEP with FSC can be observed when avoiding 2 lines in corridor 1-2 and 1-5 with better distribution of the power flow in the system rather than installing FSC or lines in them. The total investment cost of installed FSCs is US$22.8×$10^6$, but US$70.11×10^6 is saved in transmission investment. The benefit-cost ratio is 3.07, which is quite a good ratio.

Table 15 - Power Flow between Region 1 and 2 of the IEEE 24-Bus System

Transmission Lines	TEP without FSC Flow (MW)	TEP with FSC Flow (MW)	Max Flow (MW)
24-3	397	400	400
11-9	246	386	400
11-10	305	320	400
12-9	322	385	400
12-10	381	397	400
Total	1651	1888	2000

Source: The author

The processing time rapidly increases from test 1 to test 2, since in test 2, the 10 anticipated outages in transmission lines drives the number of variables and contingency dependent constraints 10 times higher than test 1. In test 3, the FSCs are also added to the problem and 5 more contingencies are anticipated in FSCs, resulting in a much longer processing time. Therefore, when the number of contingencies grows, the problems become intractable and difficult to solve. It may be possible to use other modeling techniques or methods such as heuristic approaches to solve this problem but the optimum solution cannot be guaranteed.

It is also worth mentioning that the inclusion of FSCs has significant influence on the installed devices in each planning stage. In multistage TEP with FSCs, Figure 14, no lines are added in stages 2 and 3 while in planning without FSCs, 5 lines are considered for installation in stages 2 and 3. It is also interesting to note that in test 3 new lines are installed in stage 1 and the transfer capacities of these lines are increased in stage 2 using FSCs.

5.3.2 Colombian System

This system has 93 buses, 155 corridors and three planning stages. The data and initial topology are provided in (LAPSEE, 2012) and in Appendix I. The multistage TEP planning of this system has been studied in (VINASCO et al., 2011) without contingency and FSC allocation. In this study, we also solve the problem without contingency and also without angle limitation so that the results are comparable to (VINASCO et al., 2011). The optimum solution of this system without contingency in transmission lines and without FSC allocation, also reported in (VINASCO et al., 2011), was obtained in 15 minutes with an investment of US$492,167×$10^3$ and installation of 19 transmission lines, as follows.

- Stage 1 (US$338,744×$10^3$): n_{57-81}= 2, n_{55-57}= 1, n_{55-62}= 1, n_{45-81}= 1, n_{82-85}= 1,
- Stage 2 (US$76,362×$10^3$): n_{27-29}= 1, n_{62-73}= 1, n_{72-73}= 1,
- Stage 3 (US$77,060×$10^3$): n_{19-82}=1, n_{43-88}=2, n_{15-18}=1,n_{30-65}=1, n_{30-72}=1, n_{55-84}=1, n_{27-64}= 1, n_{19-82}= 1, n_{68-86}= 1.

Considering allocation of FSC, a different transmission line topology with a lower cost is obtained in about 5 hours. The installed transmission lines and FSCs in different planning stages are shown in Figure 15. Similar to the studies of the IEEE 24-bus system, the lines installed in the first, second and third stages appear in blue, green and red, respectively. The total number of installed transmission lines is 11, and the investment cost is US$410,725×$10^3$ (US$325,745×$10^3$ for transmission lines and US$ 84,979×10^3 for FSC):

- Stage 1 (US$230,809×$10^3$): candidate lines (US$180,414×10^3) n_{55-57}=1, n_{56-57}=1, n_{55-62}=1; FSC in existing lines (US$50,394×$10^3$) $z_{14-31,3}$, $z_{45-54,3}$, $z_{66-69,3}$, $z_{9-69,3}$, $z_{60-69,3}$, $z_{32-34,3}$, $z_{31-34,3}$, $z_{19-22,3}$, $z_{19-66,3}$, $z_{4-34,3}$, $z_{67-68,3}$, $z_{19-86,2}$,
- Stage 2 (US$139,982×$10^3$): candidate lines (US$124,187×10^3) n_{52-88}=1, n_{43-88}=1, n_{59-67}=1, n_{27-64}=1, n_{62-73}=1; FSC in existing lines (US$15,795×$10^3$) $z_{35-36,3}$, $z_{47-52,3}$, $z_{47-54,3}$, $z_{19-58,3}$, $z_{64-65,3}$, $z_{39-43,3}$, $z_{50-54,3}$,
- Stage 3 (US$ 39,936×10^3): candidate lines (US$21,145×$10^3$) n_{55-84}=1; n_{68-86}=1; FSC in existing lines (US$12,950×$10^3$) $z_{14-18,3}$, $z_{14-60,2}$, $z_{30-64,3}$, $z_{16-18,3}$, $z_{18-20,3}$, $z_{18-66,3}$, $z_{41-43,2}$; and FSCs in candidate lines (US$5,840×$10^3$) $u_{43-88,3}$ and $u_{19-66,3}$.

The investment cost in transmission lines in stage 1 of multistage TEP without FSCs is much higher than planning with FSCs, hence, a significant benefit is achieved by postponing line installation to the later stages. Of the 19 lines installed in the planning without FSC, only 7 lines are installed in planning with FSC, and 12 are removed from the solution of the multistage TEP without FSC. Instead, 4 different lines and 39 FSCs (with different compensation levels) are installed. Therefore, an FSC investment of US$84,979×$10^3$ (net present value) saves US$166,422×$10^3$ in transmission investments, and the benefit-cost ratio is 1.95, again, the FSC allocation produces excellent results. The use of the transfer capacity of the whole network results in the obtained level of performance. This is illustrated in Table 16 which summarizes the transmission lines' flow with a difference greater than 80 MW in multistage TEP with and without FSC. As this table shows, the total transmission capacity for these lines is 11460 MW. From this amount, 5965 MW is used (52%) in planning without FSC, while 7878 MW is used (68%) for planning with FSC, demonstrating significant utilization of network transfer capacity.

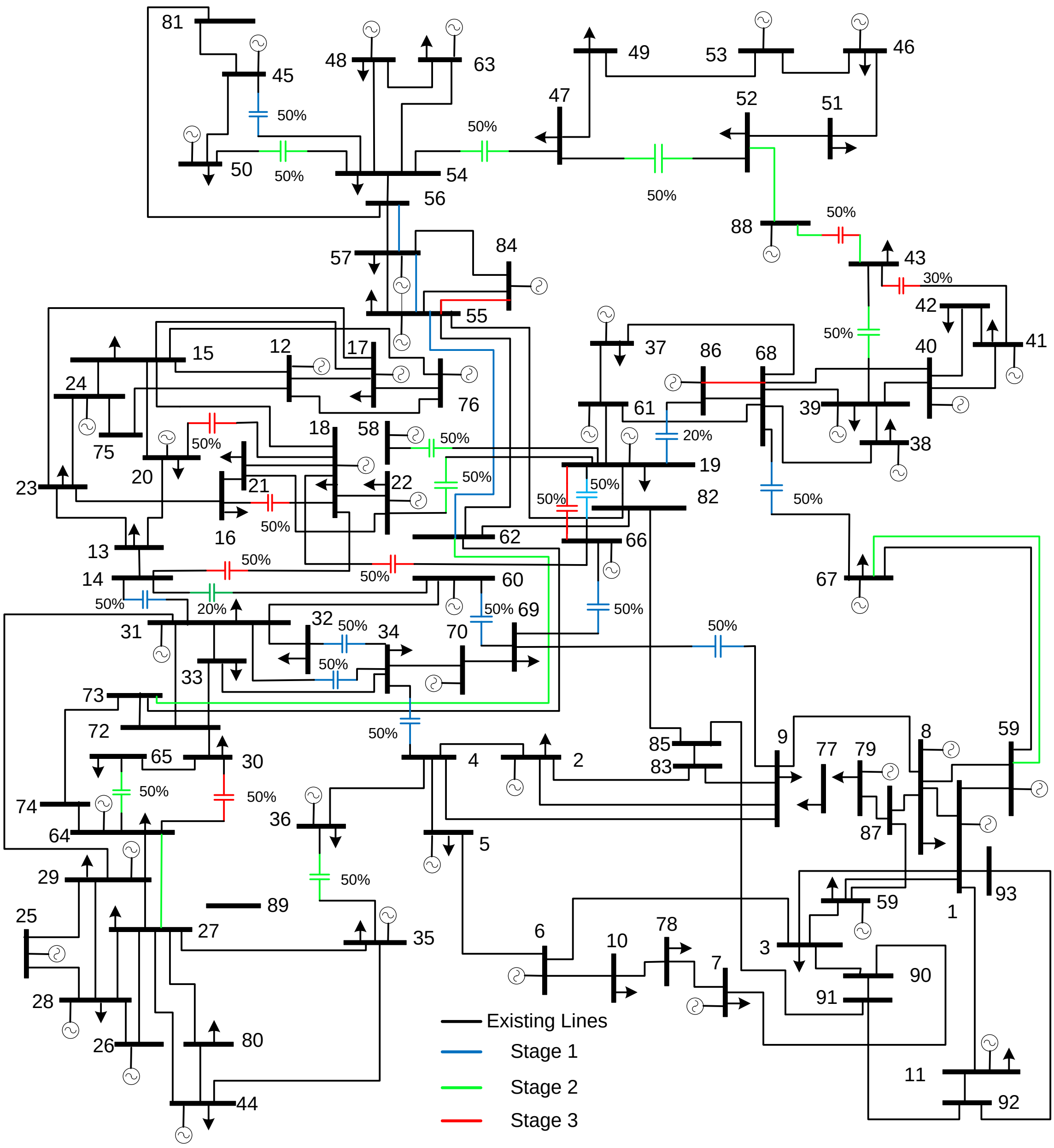
Table 16 - Power flow in transmission lines for the Colombian system

Lines	TEP without FSC Flow (MW)	TEP with FSC Flow (MW)	Max Flow (MW)	Lines	TEP without FSC Flow (MW)	TEP with FSC Flow (MW)	Max Flow (MW)
43-88	150	250*	250	16-18	211	350*	350
14-18	68	172*	250	16-23	38	166	350
14-60	212	123*	300	18-20	220	350*	350
2-83	419	266	570	18-66	155	245*	350
9-83	322	122	400	27-29	245	342	350
15-18	309	450	450	4-34	112	198*	270
56-81	27	384	550	50-54	16	120*	250
45-54	31	191*	320	54-56	254	393	450
45-50	213	316	350	72-73	304	414	500
30-65	115	202	250	83-85	370	194	450
30-72	171	267	350	90-91	513	394	550
56-57	368	521	600	85-91	541	312	600
66-69	133	250*	250	1-93	42	124	450
9-69	160	329*	350	92-93	42	124	600
60-69	204	309*	350				
Total	-	-	-	-	5965	7878	11460

* Transmission Lines having fixed series compensation

Source: The author

Figure 15 - Installed lines and FSCs for Colombian 93-bus bus system in TEP with FSC



Source: The author

CONCLUSIONS

In this thesis, the latest models for transmission expansion planning problem are provided. The complete model of AC transmission expansion planning is proposed. This model takes into consideration all parameters of a power system in planning, including phase shifter transformer, bus systems, shunt susceptances and conductance, transmission lines resistance, etc. Other transmission expansion planning models, including DC with power losses, pure DC model, disjunctive, transportation and hybrid model, for both static and multistage planning, are also discussed.

The concept of the binary numeral system is used to reduce the number of binary variables in the disjunctive model of the static and multistage transmission expansion planning problem. Based on this concept, the reduced disjunctive model is proposed. This model was tested with several test systems and presented an excellent performance in terms of computation time and memory usage.

Several fence constraints, deduced from the power balance in each node or supernode, are also proposed for the reduced disjunctive model for the fast convergence of the problem. The constraints are obtained based on a power deficit in each node or supernode of the system. Two types of constraints are proposed to be used in each system node or supernode. A very simple procedure is considered to check the efficiency of constraints.

In studies, we found that the optimum solution of some real test systems is still unreachable or that the convergence rate is very slow, even with state-of-the-art computer technology and commercial branch and bound solvers which are able to utilize all computer processors in parallel to solve the problem. Therefore, a metaheuristic based on the GRASP construction phase is proposed in order to greatly reduce in the search space of the problems. In case studies, the performance of the proposed domain reduction strategy, GRASP-CP, is evaluated using the transportation model of transmission expansion planning. These studies obtain the best solutions of Brazilian North-Northeast system for the transportation model.

The performance of the reduced disjunctive model together with fence constraints is assessed through application to several real test systems. In these tests, the optimum solution of some systems are obtained with much less effort in comparison to the common disjunctive model. The proposed methodology is also applied to solve the TEP problem of the complex Brazilian North-Northeast system. The best solutions are proposed for both the static and multistage TEP problems for this system.

In this thesis, we also propose a linear mixed binary formulation for the multistage transmission expansion planning problem considering fixed series compensations (FSCs) allocation and N-1 security constraints for both transmission lines and FSCs. This problem is of significant interest to transmission utilities and national planning bureaus and is applicable to every country and market. Redistributing active power flows with FSCs makes better use of existing and any prospective transmission lines. An excellent benefit-cost ratio can be expected from integration of FSC in multistage transmission expansion planning. Two test systems were used to show the accuracy and efficiency of the proposed model. The results in both cases have shown an economic gain obtained when FSCs are allocated through the multistage transmission expansion planning.

CONCLUSÃO DO TRABALHO

Nesta tese foram desenvolvidos novos modelos matemáticos para o problema de planejamento da expansão de sistemas de transmissão. O modelo completo AC de planejamento da expansão de sistemas de transmissão também é proposto neste trabalho. Este modelo considera todos os parâmetros de um sistema elétrico no processo de planejamento, incluindo os transformadores defasadores, as susceptâncias shunt das barras, as resistências e reatâncias das linhas de transmissão, etc. Também são discutidos em detalhe nesta tese outros modelos de planejamento da expansão de sistemas de transmissão, incluindo o modelo DC com perdas, o modelo linear disjuntivo, o modelo de transportes e o modelo híbrido para o planejamento estático e multiestágio.

É usado o conceito de sistema de numeração binária para reduzir o número de variáveis binárias no modelo linear disjuntivo no problema de planejamento da expansão de sistemas de transmissão estático e multiestágio. Baseado neste conceito, é proposto o modelo linear disjuntivo reduzido. Este modelo foi testado usando vários sistemas de testes e apresentou excelente desempenho em tempo de processamento e uso de memória.

Também é proposta a inclusão de várias restrições adicionais deduzidas usando o conceito de balanço de potência em um nó ou em um supernó o que permitiu acelerar o processo de convergência dos métodos abordados. Essas restrições são geradas baseadas no deficit ou excesso de potência em um nó ou em um supernó do sistema elétrico. Assim, são propostos dois tipos de restrições que são usadas em cada nó ou supernó do sistema. Adicionalmente, uma estratégia muito simples é considerada para verificar a eficiência dessas restrições.

Na pesquisa realizada foram encontradas as soluções ótimas de alguns sistemas reais cujas soluções ainda não eram conhecidas ou foram encontradas com uma taxa de convergência muito pequena, ajudados pelo grande desenvolvimento das tecnologias de computação e dos solvers baseados em técnicas da família branch and bound que podem usar todos os processadores de um sistema de computação em paralelo para resolver um problema. Adicionalmente, uma metaheurística baseada na lógica construtiva do GRASP foi proposta para produzir uma grande redução do espaço de busca dos problemas. Nos trabalhos desenvolvidos, a proposta de redução do espaço de busca usada, GRASP-CP, é avaliada usando o modelo de transportes do problema de planejamento da expansão de sistemas de transmissão. Nos trabalhos foram encontradas as melhores soluções conhecidas para o modelo de transportes do sistema Norte-Nordeste brasilieiro.

Foi mostrado o desempenho do modelo linear disjuntivo reduzido junto com a adição de restrições adicionais através de testes em alguns sistemas reais. Nesses testes foram encontradas as soluções ótimas de alguns sistemas com um esforço computacional menor que o modelo linear disjuntivo tradicional. A metodologia proposta também foi usada para resolver um sistema de elevada complexidade, tal como o sistema Norte-Nordeste brasileiro. Nesse caso, foram encontradas as melhores soluções conhecidas para os modelos estático e multiestágio do problema.

Nesta tese também foi proposta uma formulação linear binária mista para o problema de planejamento multiestágio da expansão de sistemas de transmissão considerando a alocação da compensação série (FSCs) e as restrições de segurança N-1 para a operação das linhas de transmissão e dos dispositivos de compensaão série. Este tipo de problema é de grande interesse para as empresas transmissoras e para os responsáveis nacionais da expansão do sistema elétrico e podem ser aplicáveis em cada país e em cada tipo de mercado. Assim, pode-se verificar que os dispositivos FSCs redistribuem de forma mais eficiente os fluxos de potência através das linhas de transmissão existentes e as adicionadas no processo de expansão. Portanto, pode-se esperar uma excelente relação custo/benefício com a integração de dispositivos FSCs no planejamento multiestágio da expansão de sistemas de transmissão. Dois testes foram realizados para mostrar a qualidade e a eficiência do modelo proposto. Em ambos os casos os resultados mostram um ganho econômico encontrado quando usamos dispositivos FSCs como elementos de alocação no processo de expansão no planejamento multiestágio da expansão de sistemas de transmissão.

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APPENDIX A. BASIC CONCEPTS

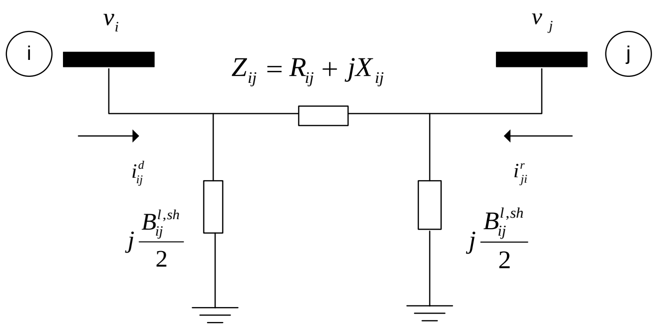
A. I. INTRODUCTION

In this appendix, we provide some fundamental basic concepts that are essential for the study and modeling of transmission expansion planning problems. The basic power system equations and shortest path problems are provided in this appendix.

A. II. POWER FLOW EQUATIONS FOR TRANSMISSION LINES

The π model of transmission line between buses *i* and *j* is presented in Figure 16, where Z_{ij} is the impedance of the line composed of line resistance, (R_{ij}), and reactance, (X_{ij}). B_{ij} is the shunt susceptance of a transmission line. In this figure, v_i and v_j are the voltage level at buses *i* and *j*, respectively.

Figure 16 - The π model for a transmission line



Source: (MONTICELLI, 1983)

Owing to power losses, the power flow in sending and receiving buses are different, and, therefore, the power flow equations for transmission lines are defined by different variables. In order to obtain the expression of the active and reactive power flow, we start with the complex power flow from bus *i* to bus *j* as given by (30).

$$s_{ij}^{d\,*} = p_{ij}^{d} - jq_{ij}^{d} = v_i^{*} i_{ij}^{d} \tag{30}$$

where i_{ij}^{d} is the electric current leaving bus *i* and given by (31) :

$$i_{ij}^{d} = Y_{ij}(v_i - v_j) + jv_i \frac{B_{ij}^{l,sh}}{2} \tag{31}$$

Y_{ij} is the admittance of the series element and calculated in (32).

$$Y_{ij} = \frac{1}{Z_{ij}} = \frac{1}{R_{ij} + jX_{ij}} = \frac{R_{ij} - jX_{ij}}{R_{ij}^2 + X_{ij}^2} = \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} - j\frac{X_{ij}}{R_{ij}^2 + X_{ij}^2} = G_{ij}^{l} + jB_{ij}^{l} \tag{32}$$

G_{ij}^{l} and B_{ij}^{l} are respectively series conductance and susceptance of the transmission line. Therefore, the complex power flow is stated as:

$$\begin{aligned} s_{ij}^{d\,*} &= v_i^{*}\left[Y_{ij}(v_i - v_j) + jv_i \frac{B_{ij}^{sh,l}}{2}\right] \\ &= Y_{ij}v_i^2 - Y_{ij}v_i^{*}v_j + j\frac{B_{ij}^{sh,l}}{2}v_i^2 \\ &= \left[G_{ij}^{l} + j(B_{ij}^{l} + j\frac{B_{ij}^{sh,l}}{2})\right]v_i^2 - (G_{ij}^{l} + jB_{ij}^{l})v_i v_j(\cos\theta_{ij} - j\sin\theta_{ij}) \end{aligned} \tag{33}$$

The active and reactive power flows are obtained by separating the real and imaginary parts of the complex power flow:

$$p_{ij}^{d} = \Re\{s_{ij}^{d}\} = G_{ij}^{l}v_i^2 - v_i v_j(G_{ij}^{l}\cos\theta_{ij} + B_{ij}^{l}\sin\theta_{ij}) \tag{34}$$

$$q_{ij}^{d} = \Im(s_{ij}^{d}) = -(B_{ij}^{l} + \frac{B_{ij}^{l,sh}}{2})v_i^2 - v_i v_j(G_{ij}^{l}\sin\theta_{ij} - B_{ij}^{l}\cos\theta_{ij}) \tag{35}$$

In the same way, active and reactive power flow leaving bus *j* toward bus *i* is given as follows:

$$p_{ji}^{r} = G_{ij}^{l}v_i^2 - v_i v_j(G_{ij}^{l}\cos\theta_{ij} - B_{ij}^{l}\sin\theta_{ij}) \tag{36}$$

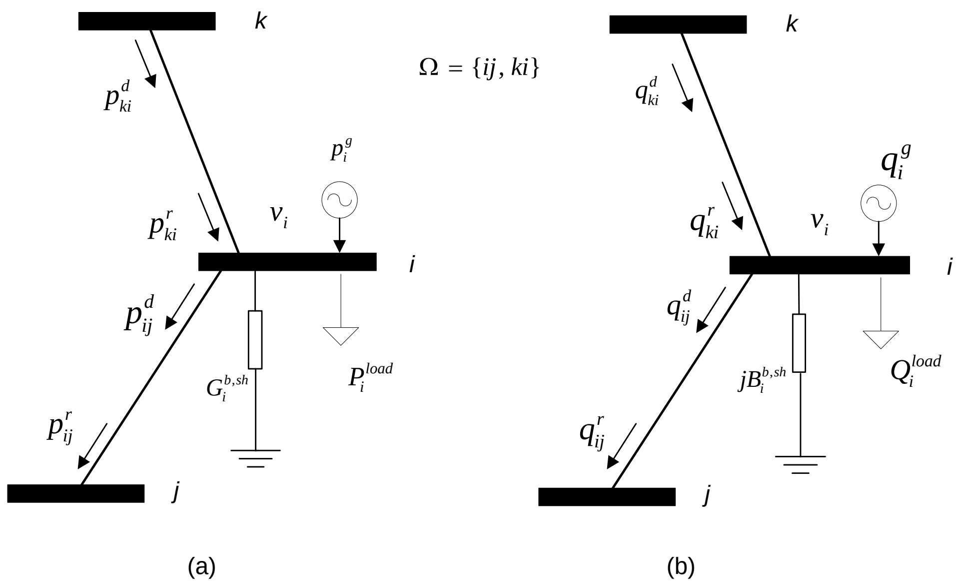
$$q_{ji}^{r} = -(B_{ij}^{l} + \frac{B_{ij}^{l,sh}}{2})v_j^2 + v_i v_j(G_{ij}^{l}\sin\theta_{ij} + B_{ij}^{l}\cos\theta_{ij}) \tag{37}$$

Note that p_{ji}^{r} and q_{ji}^{r} can also be obtained simply by changing the indexes of *i* and *j* in the expressions of p_{ij}^{d} and q_{ij}^{d} .

A. III. POWER BALANCE EQUATIONS FOR BUSES

A generic power system bus (i) with two neighboring buses (k and j) and the lines connecting them are shown in Figure 17. This figure is used to discuss active and reactive power balances for each bus. This is the exact model for the steady state active and reactive power flow of a system.

Figure 17 - The model and equations for active and reactive power in a system node



Source: The author

The active part of the power balance can be expressed using Figure 17.a, where p_i^g and P_i^d are respectively generation and load at buses i. The shunt conductance of bus i is given by $G_i^{b,sh}$. Therefore, the active power balance in bus i can be expressed by (38).

$$p_i^g - G_i^{b,sh} v_i^2 - p_{ij}^d + p_{ki}^r = P_i^{load} \tag{38}$$

In the same way, the reactive power balance can be obtained using Figure 17.b, where reactive power demand and generation are stated by Q_i^d and q_i^g, and the shunt susceptance is stated by $B_i^{b,sh}$. Hence, the power flow balance is stated by equation (39).

$$q_i^g - B_i^{b,sh} v_i^2 - q_{ij}^d + q_{ki}^r = Q_i^{load} \tag{39}$$

A. IV.SHORTEST PATH PROBLEM FOR TEP PROBLEM

The shortest path problem (SHPP) is one of the well-known problems in graph theory. In this problem, the objective is to find the shortest path between two vertices (or nodes) in a graph such that the sum of the weight of the arcs in the path is minimized.

The shortest path problem is used in transmission expansion planning to find an upper bound for the linearized power flow expression in the candidate transmission lines (BINATO et al., 2001b). In subsection 2.2.3.1, the application of the shortest path problem is shown in the disjunctive model of TEP problem. The objective of shortest path problem in TEP is to find a minimum value for the maximum angle difference between two system buses, namely *s* and *t*, where there are no direct existing lines between them.

Consider that there is a *path* through existing lines from start bus *s* to target bus *t*, that is: $path = \{\mathrm{s}k_1,\ k_1k_2,\ k_2k_3,\ \ldots,\ k_n\mathrm{t}\}$. The angle difference between buses *s* and *t* is given by:

$$\theta_s - \theta_t = (\theta_s - \theta_{k_1}) + (\theta_{k_2} - \theta_{k_3}) + \ldots + (\theta_{k_n} - \theta_t)\,. \tag{40a}$$

On the other hand, the angle difference between two given buses, namely *i* and *j*, is given by $\theta_i - \theta_j = f_{ij} X_{ij}$. f_{ij} is the lossless power flow in corridor *ij* and X_{ij} is the impedance of the corridor. Therefore, equation (40a) can be stated by (40b).

$$(\theta_s - \theta_{k_1}) + (\theta_{k_1} - \theta_{k_2}) + \ldots + (\theta_{k_n} - \theta_t) = f_{sk_1} X_{sk_1} + f_{k_1k_2} X_{k_1k_2} + f_{k_2k_n} X_{k_2k_n} + \ldots + f_{k_nt} X_{k_nt} \tag{40b}$$

Since the impedance in circuits connecting two buses is constant, the maximum value of the angle difference could not be greater than $\overline{P}_{ij} X_{ij}$, where $\overline{P}_{ij}$ is the maximum power flow in corridor *ij*. Therefore, an upper bound is obtained for the angle difference between buses *s* and *t*.

$$\theta_s - \theta_t \le \overline{P}_{sk_1} X_{sk_1} + \overline{P}_{k_1k_2} X_{k_1k_2} + \overline{P}_{k_2k_n} X_{k_2k_n} + \overline{P}_{k_nt} X_{k_nt} = \sum_{ij \in Path} \overline{P}_{ij} X_{ij} \tag{40c}$$

The *path* indicated in (40c) is an arbitrary path. In order to find the shortest path that gives the minimum value for $\sum_{ij \in Path} \overline{P}_{ij} X_{ij}$, the following model is solved:

$$\textbf{SHPP:}\ \min \sum_{ij \in (\Omega_d \cup \Omega_r)} \overline{P}_{ij} X_{ij} w_{ij} \tag{41a}$$

s.t.

$$\sum_{sj \in \Omega} w_{sj} - \sum_{js \in \Omega} w_{js} = 1 \qquad \forall j \ne t \tag{41b}$$

$$\sum_{tj\in\Omega} w_{tj} - \sum_{jt\in\Omega} w_{jt} = -1 \quad \forall j \neq s \tag{41c}$$

$$\sum_{ij\in\Omega} w_{ij} - \sum_{ji\in\Omega} w_{ji} = 0 \quad \forall i \in \beta \setminus \{s,t\} \tag{41d}$$

where $w_{ij} = 1$ means that the corridor between *i* and *j* is in the shortest path otherwise it is not in the path. Equation (41b) enforces the problem to include one of the outgoing routes from bus *s*, in the shortest path, while (41c) enforces the problem to select one of the incoming lines to bus *t*. Equation (41d) indicates that the number of incoming and outgoing routes to bus *i* ($i \neq s,\ i \neq t$) should be equal.

APPENDIX B. SYSTEMS DATA FOR THE TEP PROBLEM

In this appendix, the data for Garver system, the Southern Brazilian System, the Colombian System, and the North-Northeast Brazilian systems are provided. First, the base topology for each system presented, then, when available, the bus data for each stage or for planning with and without generation rescheduling is provided. Finally, transmission lines data are given.

B. I. SYSTEMS DATA FORMAT

The data provided in following sections are useful when the DC model of power systems is used. There are three types of data for these systems:

System Stages

1 Stage Number

2 Discount factor to find the net present value for transmission investment (%).

This data is available for systems with multistage planning in which the discount factor needed to find the net present value for the transmission investment is provided.

System Buses

1 Bus Number
2 Bus Type: 0 -> Load, 1 -> Generator, 2 -> Slack
3 Load (MW)
4 Maximum generation limit (MW)

Transmission Lines

1 Line Number

2 "From" Bus

3 "To" Bus

4 Line reactance (p.u)

5 Number of existence lines

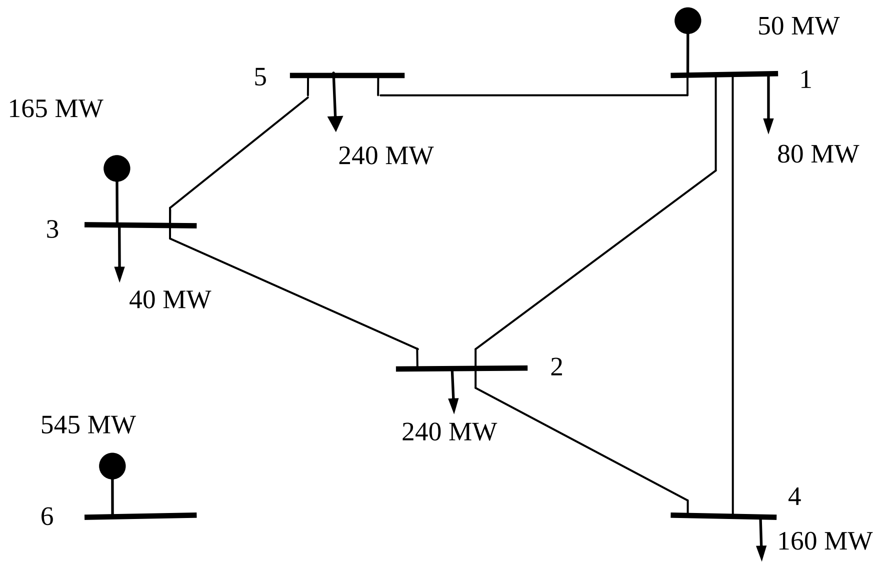
6 Maximum power flow limit in the line (MW)

7 Investment cost of the line (US$)

8 Maximum number of transmission lines in corridor

B. II. GARVER SYSTEM DATA

Figure 18 - Garver system base line



Source: The Author

Table 17 - Garver system bus data for TEP

Bus Number	Type	Load (MW)	Without Generation Rescheduling (MW)	With Generation Rescheduling (MW)
1	2	80	50	150
2	0	240	0	0
3	1	40	165	365
4	0	160	0	0
5	0	240	0	0
6	1	0	545	600

Source: (LAPSEE, 2012)

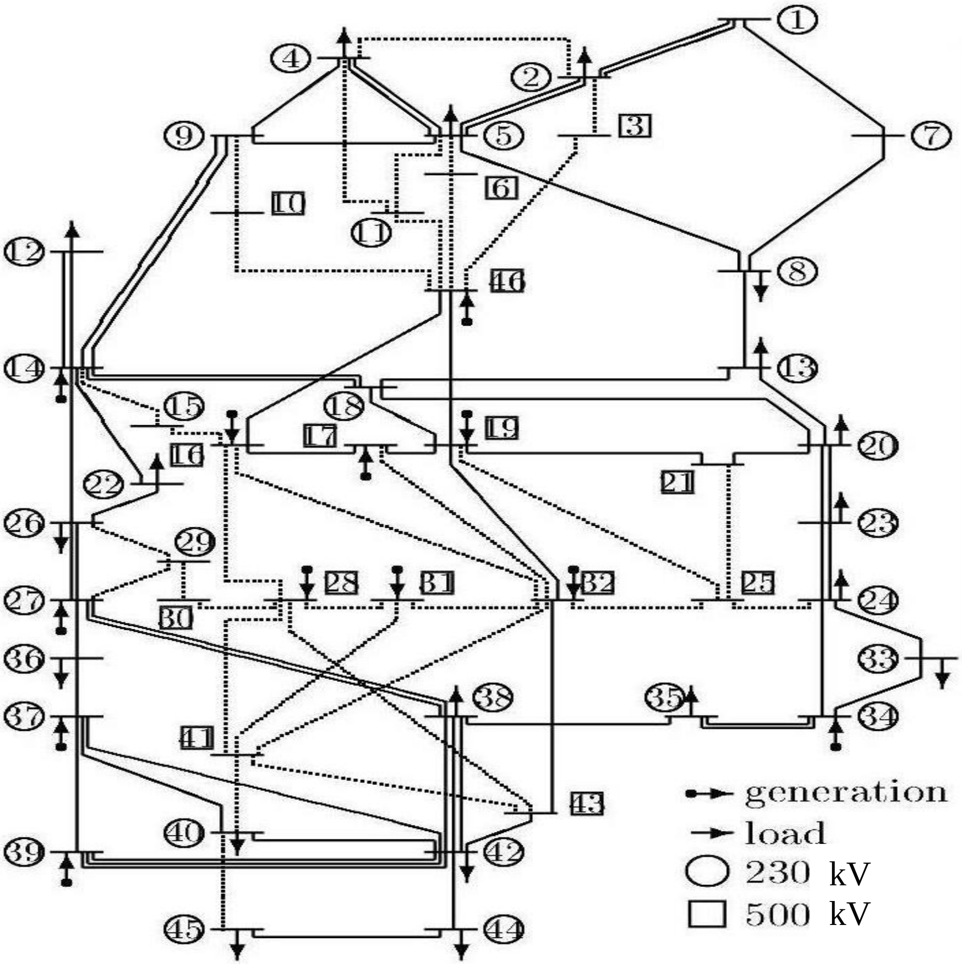
Table 18 - Garver system transmission lines data

Line No.	Line From-To	Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$)	Max. Lines
1	1-2	0.4	1	100	40	5
2	1-3	0.38	0	100	38	5
3	1-4	0.6	1	80	60	5
4	1-5	0.2	1	100	20	5
5	1-6	0.68	0	70	68	5
6	2-3	0.2	1	100	20	5
7	2-4	0.4	1	100	40	5
8	2-5	0.31	0	100	31	5
9	2-6	0.3	0	100	30	5
10	3-4	0.59	0	82	59	5
11	3-5	0.2	1	100	20	5
12	3-6	0.48	0	100	48	5
13	4-5	0.63	0	75	63	5
14	4-6	0.3	0	100	30	5
15	5-6	0.61	0	78	61	5

Source: (LAPSEE, 2012)

B. III. SOUTHERN BRAZILIAN SYSTEM DATA

Figure 19 - The Southern Brazilian system



Source: (ESCOBAR, 2002)

Table 19 - Southern Brazilian bus system data

Bus Number	Type	Load (MW)	Without Generation Rescheduling (MW)	With Generation rescheduling (MW)
1	0	0	0	0
2	0	443.1	0	0
3	0	0	0	0
4	0	300.7	0	0
5	0	238	0	0
6	0	0	0	0
7	0	0	0	0
8	0	72.2	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	511.9	0	0
13	0	185.8	0	0
14	2	0	1257	944
15	0	0	0	0
16	1	0	2000	1366
17	1	0	1050	1000
18	0	0	0	0
19	1	0	1670	773
20	0	1091.2	0	0
21	0	0	0	0
22	0	81.9	0	0
23	0	458.1	0	0
24	0	478.2	0	0
25	0	0	0	0
26	0	231.9	0	0
27	1	0	220	54
28	1	0	800	730
29	0	0	0	0
30	0	0	0	0
31	1	0	700	310
32	1	0	500	450
33	0	229.1	0	0
34	1	0	748	221
35	0	216	0	0

36	0	90.1	0	0
37	1	0	300	212
38	0	216	0	0
39	1	0	600	221
40	0	262.1	0	0
41	0	0	0	0
42	0	1607.9	0	0
43	0	0	0	0
44	0	79.1	0	0
45	0	86.7	0	0
46	1	0	700	599

Source: (LAPSEE, 2012)

Table 20 - Southern Brazilian transmission lines data

Line No.	Line From-To	Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$×$10^3$)	Max. Lines
1	1-7	0.0616	1	270	4349	3
2	1-2	0.1065	2	270	7076	3
3	4-9	0.0924	1	270	6217	3
4	5-9	0.1173	1	270	7732	3
5	5-8	0.1132	1	270	7480	3
6	7-8	0.1023	1	270	6823	3
7	4-5	0.0566	2	270	4046	3
8	2-5	0.0324	2	270	2581	3
9	8-13	0.1348	1	240	8793	3
10	9-14	0.1756	2	220	11267	3
11	12-14	0.074	2	270	5106	3
12	14-18	0.1514	2	240	9803	3
13	13-18	0.1805	1	220	11570	3
14	13-20	0.1073	1	270	7126	3
15	18-20	0.1997	1	200	12732	3
16	19-21	0.0278	1	1500	32632	3
17	16-17	0.0078	1	2000	10505	3
18	17-19	0.0061	1	2000	8715	3
19	14-26	0.1614	1	220	10409	3

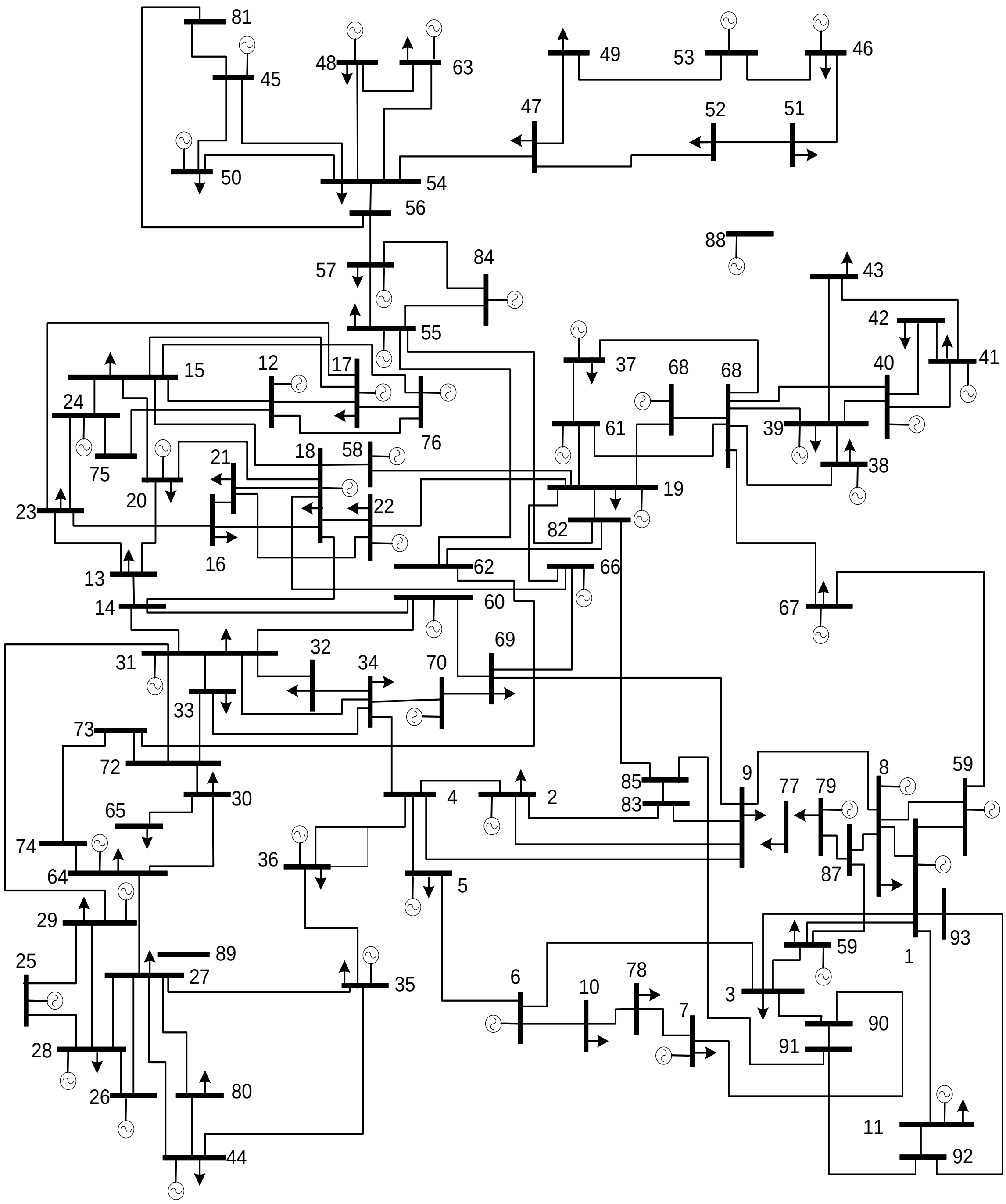
20	14-22	0.084	1	270	5712	3
21	22-26	0.079	1	270	5409	3
22	20-23	0.0932	2	270	6268	3
23	23-24	0.0774	2	270	5308	3
24	26-27	0.0832	2	270	5662	3
25	24-34	0.1647	1	220	10611	3
26	24-33	0.1448	1	240	9399	3
27	33-34	0.1265	1	270	8288	3
28	27-36	0.0915	1	270	6167	3
29	27-38	0.208	2	200	13237	3
30	36-37	0.1057	1	270	7025	3
31	34-35	0.0491	2	270	3591	3
32	35-38	0.198	1	200	12631	3
33	37-39	0.0283	1	270	2329	3
34	37-40	0.1281	1	270	8389	3
35	37-42	0.2105	1	200	13388	3
36	39-42	0.203	3	200	12934	3
37	40-42	0.0932	1	270	6268	3
38	38-42	0.0907	3	270	6116	3
39	32-43	0.0309	1	1400	35957	3
40	42-44	0.1206	1	270	7934	3
41	44-45	0.1864	1	200	11924	3
42	19-32	0.0195	1	1800	23423	3
43	46-19	0.0222	1	1800	26365	3
44	46-16	0.0203	1	1800	24319	3
45	18-19	0.0125	1	600	8178	3
46	20-21	0.0125	1	600	8178	3
47	42-43	0.0125	1	600	8178	3
48	2-4	0.0882	0	270	5965	3
49	14-15	0.0374	0	270	2884	3
50	46-10	0.0081	0	2000	10889	3
51	4-11	0.2246	0	240	14247	3
52	5-11	0.0915	0	270	6167	3
53	46-6	0.0128	0	2000	16005	3
54	46-3	0.0203	0	1800	24319	3
55	16-28	0.0222	0	1800	26365	3
56	16-32	0.0311	0	1400	36213	3
57	17-32	0.0232	0	1700	27516	3
58	19-25	0.0325	0	1400	37748	3
59	21-25	0.0174	0	2000	21121	3

60	25-32	0.0319	0	1400	37109	3
61	31-32	0.0046	0	2000	7052	3
62	28-31	0.0053	0	2000	7819	3
63	28-30	0.0058	0	2000	8331	3
64	27-29	0.0998	0	270	6672	3
65	26-29	0.0541	0	270	3894	3
66	28-41	0.0339	0	1300	39283	3
67	28-43	0.0406	0	1200	46701	3
68	31-41	0.0278	0	1500	32632	3
69	32-41	0.0309	0	1400	35957	3
70	41-43	0.0139	0	2000	17284	3
71	40-45	0.2205	0	180	13994	3
72	15-16	0.0125	0	600	8178	3
73	46-11	0.0125	0	600	8178	3
74	24-25	0.0125	0	600	8178	3
75	29-30	0.0125	0	600	8178	3
76	40-41	0.0125	0	600	8178	3
77	2-3	0.0125	0	600	8178	3
78	5-6	0.0125	0	600	8178	3
79	9-10	0.0125	0	600	8178	3

Source: (LAPSEE, 2012)

B. IV. COLOMBIAN SYSTEM DATA

Figure 20 - Colombian 93 bus system initial topology



Source: The author

Colombian system planning stages:

Stage	Discount factor
1	1
2	0.729
3	0.478

Table 21 - Colombian system bus data for three planning stages

Bus Number	Type	Load (2005) (MW)	Generation (2005) (MW)	Load (2009) (MW)	Generation (2009) (MW)	Load (2012) (MW)	Generation (2012) (MW)
1	1	0	241	0	241	0	241
2	0	352.9	0	406.53	165	486.66	165
3	0	393	0	490.5	0	587.08	0
4	0	0	0	0	0	0	0
5	1	235	40	293.56	40	351.42	40
6	1	0	34	0	34	0	34
7	0	300	0	374.26	0	448.03	136
8	1	339	100	423	230	505.87	230
9	0	348	0	434.12	0	519.69	0
10	0	60	0	74.21	0	88.84	0
11	1	147	80	183.9	108	220.15	108
12	1	0	47	0	47	0	47
13	0	174	0	217.26	0	260.08	0
14	0	0	0	0	0	0	0
15	0	377	0	470.17	0	562.84	0
16	0	236	0	294	0	351.9	0
17	1	136	35	169.57	35	203	35
18	1	36.2	480	45.2	540	54.1	539
19	1	19.6	900	24.46	1340	29.28	1340
20	0	202.4	0	252.5	0	302.27	45
21	0	186	0	231.7	0	277.44	0
22	1	53	200	66.13	200	79.17	200
23	0	203	0	252.5	0	302.27	0
24	1	0	120	0	150	0	150
25	1	0	86	0	86	0	86
26	1	0	70	0	70	0	70

27	0	266	0	331.4	0	396.71	0
28	0	326	0	406.3	0	486.39	14
29	1	339	618	422.6	617	505.96	618
30	0	137	0	166.7	0	199.55	0
31	1	234	189	327.3	189	391.88	189
32	0	126	0	157.3	0	188.33	0
33	0	165	0	206.53	0	247.24	0
34	0	77.5	0	96.7	0	115.81	0
35	1	172	200	214.6	200	256.86	200
36	0	112	0	140	0	167.29	44
37	1	118	138	147.3	138	176.3	138
38	0	86	0	108.4	15	129.72	15
39	0	180	0	224	0	268.19	15
40	1	0	305	0	305	0	305
41	1	54.8	70	68.4	100	81.85	100
42	0	102	0	127.3	0	152.39	0
43	0	35.4	0	44.2	0	52.9	0
44	1	257	23	321.3	23	384.64	23
45	1	0	950	0	1208	0	1208
46	1	121	150	151.7	150	181.62	150
47	0	41.15	0	51.5	0	61.6	0
48	1	600	775	750	885	896.26	885
49	0	130	0	162	0	193.27	0
50	1	424	240	528	240	632.75	240
51	0	128	0	159	0	190.45	0
52	0	38	0	46.5	0	55.6	0
53	1	0	280	0	320	0	320
54	0	76	0	95.3	0	114.19	0
55	1	223	39	279	40	333.59	40
56	0	0	0	0	0	0	0
57	0	226	0	281	130	336.94	130
58	1	0	190	0	190	0	190
59	1	0	160	0	160	0	160
60	2	0	1191	0	1216	0	1216
61	1	0	155	0	155	0	155
62	0	0	0	0	0	0	0
63	1	35	900	44	1090	52.77	1090
64	0	88	0	110.55	0	132.35	280
65	0	132	0	165	0	197.58	0
66	1	0	200	0	300	0	300

67	1	266	474	332.45	474	397.98	474
68	0	0	0	0	0	0	0
69	0	71.4	0	89	0	106.61	0
70	1	0	30	0	180	0	180
71	0	315	0	393	211	471.21	424
72	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0
76	1	0	40	0	40	0	40
77	0	55	0	70	0	82.85	0
78	0	36.65	0	45.1	0	54.07	0
79	0	98	0	123	0	146.87	300
80	0	60	0	72	0	88.34	0
81	0	0	0	0	0	0	0
82	0	0	0	0	0	0	0
83	0	0	0	0	0	0	0
84	0	0	0	0	0	0	500
85	0	0	0	0	0	0	0
86	0	0	0	0	300	0	850
87	0	0	0	0	0	0	0
88	0	0	0	0	0	0	300
89	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0
93	0	0	0	0	0	0	0

Source: (LAPSEE, 2012)

Table 22 - Colombian System transmission lines data

Line No.	Line From-To	Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$×$10^6$)	Max. Lines
1	52-88	0.098	0	300	34.19	5
2	43-88	0.1816	0	250	39.56	5
3	57-81	0.0219	0	550	58.89	5
4	73-82	0.0374	0	550	97.96	5
5	27-89	0.0267	0	450	13.27	5
6	74-89	0.0034	0	550	14.57	5

7	73-89	0.0246	0	550	66.65	5
8	79-83	0.0457	0	350	15.4	5
9	8-67	0.224	0	250	29.2	5
10	39-86	0.0545	0	350	9.88	5
11	25-28	0.0565	1	320	9.77	5
12	25-29	0.057	1	320	9.88	5
13	13-14	0.0009	2	350	3.9	5
14	13-20	0.0178	1	350	5.74	5
15	13-23	0.0277	1	350	7.01	5
16	14-31	0.1307	2	250	18.62	5
17	14-18	0.1494	2	250	20.23	5
18	14-60	0.1067	2	300	15.98	5
19	2-4	0.0271	2	350	6.66	5
20	2-9	0.0122	1	350	5.28	5
21	2-83	0.02	1	570	5.97	5
22	9-83	0.02	1	400	5.97	5
23	15-18	0.0365	1	450	7.93	5
24	15-17	0.0483	1	320	9.42	5
25	15-20	0.0513	1	320	9.65	5
26	15-76	0.0414	1	320	9.88	5
27	15-24	0.0145	1	350	5.28	5
28	37-61	0.0139	1	350	4.94	5
29	19-61	0.1105	2	250	16.09	5
30	61-68	0.0789	1	250	12.41	5
31	37-68	0.0544	1	320	9.65	5
32	40-68	0.132	1	320	18.16	5
33	12-75	0.0641	1	320	11.49	5
34	24-75	0.0161	1	350	5.51	5
35	35-36	0.2074	1	250	27.36	5
36	27-35	0.1498	1	250	22.07	5
37	35-44	0.1358	2	250	20.35	5
38	38-68	0.0389	1	350	7.93	5
39	38-39	0.03	1	350	6.32	5
40	27-80	0.0242	1	350	7.01	5
41	44-80	0.1014	1	250	17.59	5
42	56-81	0.0114	1	550	32.86	5
43	45-54	0.0946	1	320	13.56	5
44	45-50	0.007	2	350	4.36	5
45	10-78	0.0102	1	350	4.94	5
46	7-78	0.0043	1	350	4.13	5

47	30-64	0.1533	1	250	20.58	5
48	30-65	0.091	1	250	13.68	5
49	30-72	0.0173	2	350	5.51	5
50	55-57	0.0174	1	600	46.81	5
51	57-84	0.0087	1	600	26.66	5
52	55-84	0.0087	1	600	26.66	5
53	56-57	0.024	2	600	62.62	5
54	9-77	0.019	1	350	5.86	5
55	77-79	0.0097	1	350	5.17	5
56	1-59	0.0232	2	350	6.2	5
57	59-67	0.118	2	250	16.67	5
58	8-59	0.1056	2	250	15.4	5
59	1-3	0.104	1	250	15.86	5
60	3-71	0.0136	1	450	5.17	5
61	3-6	0.0497	1	350	9.42	5
62	55-62	0.0281	1	550	70.99	5
63	47-52	0.0644	1	350	10.57	5
64	51-52	0.0859	1	250	12.87	5
65	29-31	0.1042	2	250	32.98	5
66	41-42	0.0094	1	350	4.71	5
67	40-42	0.0153	1	350	5.17	5
68	46-53	0.1041	2	250	14.6	5
69	46-51	0.1141	1	250	16.32	5
70	69-70	0.0228	2	350	6.2	5
71	66-69	0.1217	2	250	17.13	5
72	9-69	0.1098	2	350	15.75	5
73	60-69	0.0906	2	350	13.68	5
74	31-32	0.0259	1	350	6.55	5
75	32-34	0.054	1	350	9.77	5
76	16-18	0.0625	1	350	10.92	5
77	16-23	0.0238	1	350	6.89	5
78	16-21	0.0282	1	350	6.89	5
79	31-34	0.0792	1	250	12.41	5
80	31-33	0.0248	2	350	6.43	5
81	31-60	0.1944	2	250	25.98	5
82	31-72	0.0244	2	350	6.32	5
83	47-54	0.1003	2	250	14.25	5
84	47-49	0.0942	2	250	13.56	5
85	18-58	0.0212	2	350	5.74	5
86	18-20	0.0504	1	350	9.54	5

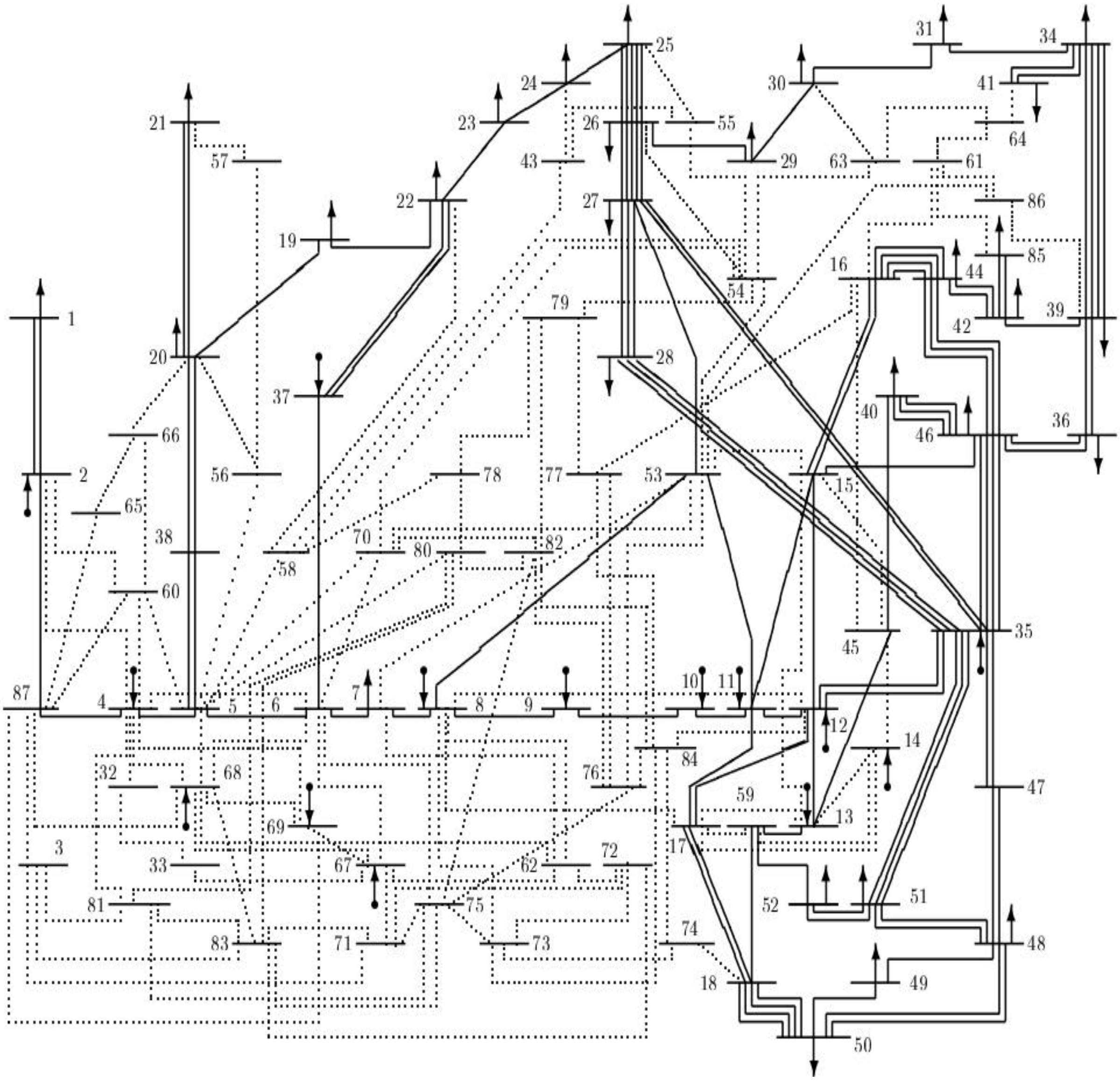
87	18-66	0.0664	2	350	11.38	5
88	18-21	0.0348	1	350	7.47	5
89	18-22	0.0209	1	350	6.43	5
90	19-22	0.0691	1	350	11.72	5
91	4-5	0.0049	3	350	4.25	5
92	5-6	0.0074	2	350	4.48	5
93	17-23	0.0913	1	250	12.99	5
94	17-76	0.002	1	350	3.9	5
95	12-17	0.0086	1	350	4.71	5
96	1-71	0.0841	2	250	14.37	5
97	1-8	0.081	1	250	13.22	5
98	1-11	0.0799	1	250	12.53	5
99	4-36	0.085	2	250	13.56	5
100	19-58	0.0826	1	320	11.72	5
101	27-64	0.028	1	350	6.78	5
102	27-28	0.0238	1	350	6.2	5
103	27-44	0.0893	1	250	16.32	5
104	26-27	0.0657	1	350	10.92	5
105	27-29	0.0166	1	350	5.05	5
106	19-66	0.0516	1	350	9.31	5
107	73-74	0.0214	1	600	58.28	5
108	64-65	0.0741	1	350	11.84	5
109	29-64	0.0063	1	350	4.36	5
110	4-34	0.1016	2	270	14.94	5
111	34-70	0.0415	2	350	8.27	5
112	33-34	0.1139	1	320	16.32	5
113	8-71	0.0075	1	400	4.48	5
114	54-63	0.0495	3	320	9.08	5
115	48-63	0.0238	1	350	6.32	5
116	67-68	0.166	2	250	22.07	5
117	39-68	0.0145	1	350	5.28	5
118	8-9	0.0168	1	350	5.97	5
119	79-87	0.0071	1	350	4.48	5
120	8-87	0.0132	1	350	5.17	5
121	39-43	0.1163	1	250	16.55	5
122	41-43	0.1142	1	250	16.32	5
123	23-24	0.0255	1	350	6.32	5
124	21-22	0.0549	1	350	9.88	5
125	26-28	0.0512	1	350	9.31	5
126	28-29	0.0281	1	350	6.78	5

127	6-10	0.0337	1	350	7.58	5
128	33-72	0.0228	1	350	6.2	5
129	39-40	0.102	2	250	16.21	5
130	12-76	0.0081	1	350	4.71	5
131	48-54	0.0396	3	350	8.04	5
132	50-54	0.0876	2	250	12.87	5
133	62-73	0.0272	1	750	73.16	5
134	49-53	0.1008	2	250	14.25	5
135	40-41	0.0186	1	350	5.74	5
136	45-81	0.0267	1	450	13.27	5
137	64-74	0.0267	1	500	13.27	5
138	54-56	0.0267	3	450	13.27	5
139	60-62	0.0257	3	450	13.27	5
140	72-73	0.0267	2	500	13.27	5
141	19-82	0.0267	1	450	13.27	5
142	55-82	0.029	1	550	77.5	5
143	62-82	0.0101	1	600	31	5
144	83-85	0.0267	2	450	13.27	5
145	82-85	0.0341	1	700	89.9	5
146	19-86	0.1513	1	300	20.92	5
147	68-86	0.0404	1	350	8.27	5
148	7-90	0.005	2	350	4.25	5
149	3-90	0.0074	1	350	4.59	5
150	90-91	0.0267	1	550	13.27	5
151	85-91	0.0139	1	600	40.3	5
152	11-92	0.0267	1	450	13.27	5
153	1-93	0.0267	1	450	13.27	5
154	92-93	0.0097	1	600	30.07	5
155	91-92	0.0088	1	600	27.59	5

Source: (LAPSEE, 2012)

B. V. BRAZILIAN NORTH-NORTHEAST SYSTEM DATA

Figure 21 - Brazilian North-Northeast system



Source: (ESCOBAR, 2002)

Brazilian North-Northeast system planning stages:

Stage	Discount factor
1	1
2	0.656

Table 23 - Brazilian North-Northeast system bus data

Bus Number	Type	Load (2002) (MW)	Generation (2002) (MW)	Load (2008) (MW)	Generation (2008) (MW)
1	0	1857	0	2747	0
2	1	0	4048	0	4550
3	0	0	0	0	0
4	1	0	517	0	6422
5	0	0	0	0	0
6	0	0	0	0	0
7	0	31	0	31	0
8	1	0	403	0	82
9	1	0	465	0	465
10	1	0	538	0	538
11	1	0	2200	0	2260
12	1	0	2257	0	4312
13	2	0	4510	0	5900
14	1	0	542	0	542
15	0	0	0	0	0
16	0	0	0	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	86	0	125	0
20	0	125	0	181	0
21	0	722	0	1044	0
22	0	291	0	446	0
23	0	58	0	84	0
24	0	159	0	230	0

25	0	1502	0	2273	0
26	0	47	0	68	0
27	0	378	0	546	0
28	0	189	0	273	0
29	0	47	0	68	0
30	0	189	0	273	0
31	0	110	0	225	0
32	0	0	0	0	0
33	0	0	0	0	0
34	0	28	0	107	0
35	1	0	1635	0	1531
36	0	225	0	325	0
37	1	0	169	0	114
38	0	0	0	0	0
39	0	186	0	269	0
40	0	1201	0	1738	0
41	0	520	0	752	0
42	0	341	0	494	0
43	0	0	0	0	0
44	0	4022	0	5819	0
45	0	0	0	0	0
46	0	205	0	297	0
47	0	0	0	0	0
48	0	347	0	432	0
49	0	777	0	1124	0
50	0	5189	0	7628	0
51	0	290	0	420	0
52	0	707	0	1024	0
53	0	0	0	0	0
54	0	0	0	0	0
55	0	0	0	0	0
56	0	0	0	0	0
57	0	0	0	0	0
58	0	0	0	0	0
59	0	0	0	0	0
60	0	0	0	0	0
61	0	0	0	0	0
62	0	0	0	0	0
63	0	0	0	0	0
64	0	0	0	0	0

65	0	0	0	0	0
66	0	0	0	0	0
67	1	0	1242	0	1242
68	1	0	888	0	888
69	1	0	902	0	902
70	0	0	0	0	0
71	0	0	0	0	0
72	0	0	0	0	0
73	0	0	0	0	0
74	0	0	0	0	0
75	0	0	0	0	0
76	0	0	0	0	0
77	0	0	0	0	0
78	0	0	0	0	0
79	0	0	0	0	0
80	0	0	0	0	0
81	0	0	0	0	0
82	0	0	0	0	0
83	0	0	0	0	0
84	0	0	0	0	0
85	0	487	0	705	0
86	0	0	0	0	0
87	0	0	0	0	0

Source: (LAPSEE, 2012)

Table 24 - Brazilian North-Northeastern transmission lines data

Line No.	Line From-To			Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$×$10^3$)	Max. Lines
1	1	-	2	0.0374	2	1000	44056	16
2	2	-	4	0.0406	0	1000	48880	16
3	2	-	60	0.0435	0	1000	52230	16
4	2	-	87	0.0259	1	1000	31192	16
5	3	-	71	0.0078	0	3200	92253	16
6	3	-	81	0.0049	0	3200	60153	16
7	3	-	83	0.0043	0	3200	53253	16
8	3	-	87	0.0058	0	1200	21232	16
9	4	-	5	0.0435	1	1000	52230	16
10	4	-	6	0.0487	0	1000	58260	16

11	4	-	32	0.0233	0	300	7510	16
12	4	-	60	0.0215	0	1000	26770	16
13	4	-	68	0.007	0	1000	10020	16
14	4	-	69	0.0162	0	1000	20740	16
15	4	-	81	0.0058	0	1200	21232	16
16	4	-	87	0.0218	1	1000	26502	16
17	5	-	6	0.0241	1	1000	29852	16
18	5	-	38	0.0117	2	600	8926	16
19	5	-	56	0.0235	0	1000	29182	16
20	5	-	58	0.022	0	1000	27440	16
21	5	-	60	0.0261	0	1000	32130	16
22	5	-	68	0.0406	0	1000	48880	16
23	5	-	70	0.0464	0	1000	55580	16
24	5	-	80	0.0058	0	1200	21232	16
25	6	-	7	0.0288	1	1000	35212	16
26	6	-	37	0.0233	1	300	7510	16
27	6	-	67	0.0464	0	1000	55580	16
28	6	-	68	0.0476	0	1000	56920	16
29	6	-	70	0.0371	0	1000	44860	16
30	6	-	75	0.0058	0	1200	21232	16
31	7	-	8	0.0234	1	1000	29048	16
32	7	-	53	0.0452	0	1000	54240	16
33	7	-	62	0.0255	0	1000	31460	16
34	8	-	9	0.0186	1	1000	23420	16
35	8	-	12	0.0394	0	1000	47540	16
36	8	-	17	0.0447	0	1000	53570	16
37	8	-	53	0.0365	1	1200	44190	16
38	8	-	62	0.0429	0	1000	51560	16
39	8	-	73	0.0058	0	1200	21232	16
40	9	-	10	0.0046	1	1000	7340	16
41	10	-	11	0.0133	1	1000	17390	16
42	11	-	12	0.0041	1	1200	6670	16
43	11	-	15	0.0297	1	1200	36284	16
44	11	-	17	0.0286	1	1200	35078	16
45	11	-	53	0.0254	1	1000	31326	16
46	12	-	13	0.0046	1	1200	7340	16
47	12	-	15	0.0256	1	1200	31594	16
48	12	-	17	0.0246	1	1200	30388	16
49	12	-	35	0.0117	2	600	8926	16
50	12	-	84	0.0058	0	1200	21232	16

51	13	-	14	0.0075	0	1200	10690	16
52	13	-	15	0.0215	0	1200	26770	16
53	13	-	17	0.0232	0	1200	28780	16
54	13	-	45	0.029	1	1200	35480	16
55	13	-	59	0.0232	1	1200	28780	16
56	14	-	17	0.0232	0	1200	28780	16
57	14	-	45	0.0232	0	1200	28780	16
58	14	-	59	0.0157	0	1200	20070	16
59	15	-	16	0.0197	2	1200	24760	16
60	15	-	45	0.0103	0	1200	13906	16
61	15	-	46	0.0117	1	600	8926	16
62	15	-	53	0.0423	0	1000	50890	16
63	16	-	44	0.0117	4	600	8926	16
64	16	-	45	0.022	0	1200	27440	16
65	16	-	61	0.0128	0	1000	16720	16
66	16	-	77	0.0058	0	1200	21232	16
67	17	-	18	0.017	2	1200	21678	16
68	17	-	59	0.017	0	1200	21678	16
69	18	-	50	0.0117	4	600	8926	16
70	18	-	59	0.0331	1	1200	40170	16
71	18	-	74	0.0058	0	1200	21232	16
72	19	-	20	0.0934	1	170	5885	16
73	19	-	22	0.1877	1	170	11165	16
74	20	-	21	0.0715	1	300	6960	16
75	20	-	21	0.1032	1	170	6435	16
76	20	-	38	0.1382	2	300	12840	16
77	20	-	56	0.0117	0	600	8926	16
78	20	-	66	0.2064	0	170	12210	16
79	21	-	57	0.0117	0	600	8926	16
80	22	-	23	0.1514	1	170	9130	16
81	22	-	37	0.2015	2	170	11935	16
82	22	-	58	0.0233	0	300	7510	16
83	23	-	24	0.1651	1	170	9900	16
84	24	-	25	0.2153	1	170	12705	16
85	24	-	43	0.0233	0	300	7510	16
86	25	-	26	0.1073	2	300	29636	16
87	25	-	26	0.1691	3	170	10120	16
88	25	-	55	0.0117	0	600	8926	16
89	26	-	27	0.1404	2	300	25500	16
90	26	-	27	0.2212	3	170	12760	16

91	26	-	29	0.1081	1	170	6710	16
92	26	-	54	0.0117	0	600	8926	16
93	27	-	28	0.0826	3	170	5335	16
94	27	-	35	0.1367	2	300	25000	16
95	27	-	53	0.0117	1	600	8926	16
96	28	-	35	0.1671	3	170	9900	16
97	29	-	30	0.0688	1	170	4510	16
98	30	-	31	0.0639	1	170	4235	16
99	30	-	63	0.0233	0	300	7510	16
100	31	-	34	0.1406	1	170	8525	16
101	32	-	33	0.1966	0	170	11660	16
102	33	-	67	0.0233	0	300	7510	16
103	34	-	39	0.116	2	170	7150	16
104	34	-	39	0.2968	2	80	6335	16
105	34	-	41	0.0993	2	170	6215	16
106	35	-	46	0.2172	4	170	12705	16
107	35	-	47	0.1327	2	170	8085	16
108	35	-	51	0.1602	3	170	9625	16
109	36	-	39	0.1189	2	170	7315	16
110	36	-	46	0.0639	2	170	4235	16
111	39	-	42	0.0973	1	170	6105	16
112	39	-	86	0.0233	0	300	7510	16
113	40	-	45	0.0117	1	600	8926	16
114	40	-	46	0.0875	3	170	5500	16
115	41	-	64	0.0233	0	300	7510	16
116	42	-	44	0.0698	2	170	4565	16
117	42	-	85	0.0501	2	170	3465	16
118	43	-	55	0.0254	0	1000	31326	16
119	43	-	58	0.0313	0	1000	38160	16
120	44	-	46	0.1671	3	170	10010	16
121	47	-	48	0.1966	2	170	11660	16
122	48	-	49	0.0757	1	170	4895	16
123	48	-	50	0.0256	2	170	2090	16
124	48	-	51	0.2163	2	170	12760	16
125	49	-	50	0.0835	1	170	5335	16
126	51	-	52	0.056	2	170	3795	16
127	52	-	59	0.0117	1	600	8926	16
128	53	-	54	0.027	0	1000	32120	16
129	53	-	70	0.0371	0	1000	44860	16
130	53	-	76	0.0058	0	1200	21232	16

131	53	-	86	0.0389	0	1000	46870	16
132	54	-	55	0.0206	0	1000	25028	16
133	54	-	58	0.051	0	1000	60940	16
134	54	-	63	0.0203	0	1000	25430	16
135	54	-	70	0.036	0	1000	43520	16
136	54	-	79	0.0058	0	1200	21232	16
137	56	-	57	0.0122	0	1000	16050	16
138	58	-	78	0.0058	0	1200	21232	16
139	60	-	66	0.0233	0	300	7510	16
140	60	-	87	0.0377	0	1000	45530	16
141	61	-	64	0.0186	0	1000	23420	16
142	61	-	85	0.0233	0	300	7510	16
143	61	-	86	0.0139	0	1000	18060	16
144	62	-	67	0.0464	0	1000	55580	16
145	62	-	68	0.0557	0	1000	66300	16
146	62	-	72	0.0058	0	1200	21232	16
147	63	-	64	0.029	0	1000	35480	16
148	65	-	66	0.3146	0	170	18260	16
149	65	-	87	0.0233	0	300	7510	16
150	67	-	68	0.029	0	1000	35480	16
151	67	-	69	0.0209	0	1000	26100	16
152	67	-	71	0.0058	0	1200	21232	16
153	68	-	69	0.0139	0	1000	18060	16
154	68	-	83	0.0058	0	1200	21232	16
155	68	-	87	0.0186	0	1000	23240	16
156	69	-	87	0.0139	0	1000	18060	16
157	70	-	82	0.0058	0	1200	21232	16
158	71	-	72	0.0108	0	3200	125253	16
159	71	-	75	0.0108	0	3200	125253	16
160	71	-	83	0.0067	0	3200	80253	16
161	72	-	73	0.01	0	3200	116253	16
162	72	-	83	0.013	0	3200	149253	16
163	73	-	74	0.013	0	3200	149253	16
164	73	-	75	0.013	0	3200	149253	16
165	73	-	84	0.0092	0	3200	107253	16
166	74	-	84	0.0108	0	3200	125253	16
167	75	-	76	0.0162	0	3200	185253	16
168	75	-	81	0.0113	0	3200	131253	16
169	75	-	82	0.0086	0	3200	101253	16
170	75	-	83	0.0111	0	3200	128253	16

171	76	-	77	0.013	0	3200	149253	16
172	76	-	82	0.0086	0	3200	101253	16
173	76	-	84	0.0059	0	3200	70953	16
174	77	-	79	0.0151	0	3200	173253	16
175	77	-	84	0.0115	0	3200	132753	16
176	78	-	79	0.0119	0	3200	137253	16
177	78	-	80	0.0051	0	3200	62253	16
178	79	-	82	0.0084	0	3200	98253	16
179	80	-	81	0.0101	0	3200	117753	16
180	80	-	82	0.0108	0	3200	125253	16
181	80	-	83	0.0094	0	3200	110253	16
182	81	-	83	0.0016	0	3200	23253	16
183	82	-	84	0.0135	0	3200	155253	16

Source: (LAPSEE, 2012)