Chapter 2

The Power Transfer

2.1 Introduction

The primary function of an electrical system is to supply electric power to the consumers (loads) from the sources (generators) throughout a transmission system. There are some physical properties associated to the transmission system that limitpower transfer [46][47], in spite of the capability of the generator or the requirement of the load.

Transmission systems are designed to operate according to specific voltage levels. Depending on the characteristic of the transferred power, the voltage at the transmission line ends, for instance, can be either below or above certain limits, modifying the system capacity to transfer power. Regardless the type of limit violation (up or down), actions are frequently taken to recover the assigned voltage levels, allowing the system to attend to the power demand at adequate operating condition [48].

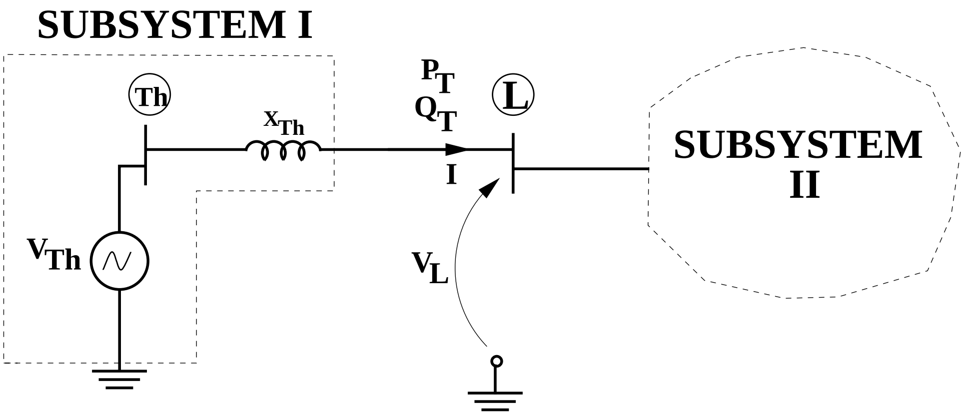
The physical parameters of transmission lines, which depend upon the line length and voltage level, strongly restrain power transfer. Series and shunt compensations have been traditionally used to modify the natural parameters of transmission lines [49].

In this Chapter, limits of power transfer due to the parameters of the transmission system are reviewed. The concepts of capability of transmission lines, which are the most numerous among the components of a transmission system, are discussed. Reactive compensation for improving power transfer is also presented.

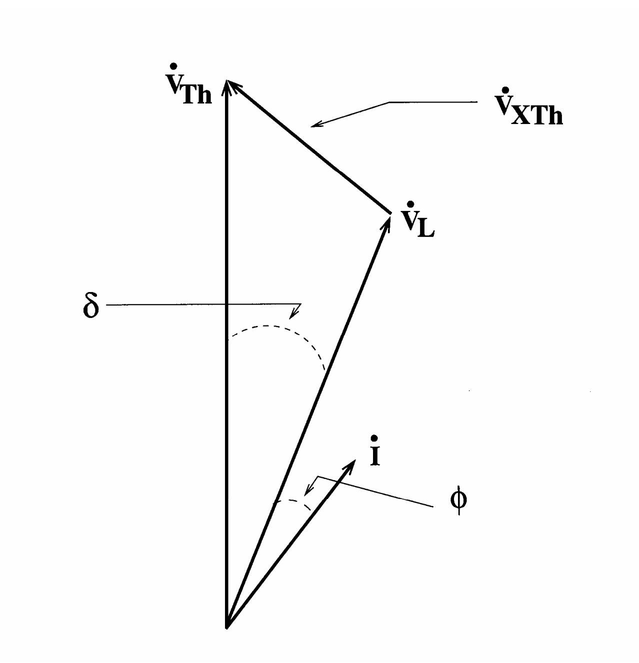
2.2 Power Transfer (P-V) Characteristic: Definition

The characteristic of power transfer (P-V characteristic) relates the voltage at the receiving-end busbar to the active power reaching it, for a given sending-end voltage, power factor and impedance of transference. The impedance of transference comprises transmission lines, transformers and other shunt and series electrical components that connect two busbars or, more generally, two subsystems.

To illustrate the power transfer characteristic, a simplified system is presented in figure 2.1a. The system is composed of a single voltage source that transfers power to a load through an impedance. The voltage source, identified as V_{Th}, represents the Thévenin's voltage of a more complex system (Subsystem I). This voltage also sets the reference for the angle of the voltage vectors. The impedance of transference between the sending and the receiving ends (busbar Th and busbar L, respectively) is named x_{Th}. A second subsystem (Subsystem II) is connected to busbar L. The active power is presently assumed to flow from Subsystem I to Subsystem II. In general, the active power can flow in either direction, depending on the power sources available in both subsystems. The active and reactive powers reaching busbar L through x_{Th} are denominated P_T and Q_T, respectively. The ***transfer power factor*** at busbar L results from the ratio between P_T and Q_T. Transfer power factor is defined as $pf_T = \cos(\phi)$, with $\phi = tg^{-1}(Q_T/P_T)$ denominated hereinafter as ***load angle***.



(a) Circuit diagram



(b) Vector diagram

Figure 2.1 Simplified system for power transfer characteristic analysis

The reactive power injected at both x_{Th} ends must satisfy the total reactive power absorbed by the impedance of transference and load. Using the load convention, the reactive power reaching a busbar through the impedance of transference is *positive* if the system connected to this busbar is *absorbing* reactive power.

In this case, pf_T is defined as *lagging* transfer power factor. The reactive power absorbed by the transmission lines is positive. Conversely, the reactive power is *negative* if the system connected to a busbar is *supplying* reactive power. A negative reactive power results in a *leading* transfer power factor. The reactive power injected into the ac system by the capacitance of long transmission lines, for instance, is negative.

In figure 2.1a, if the Subsystem II is absorbing reactive power, Q_T is positive. The total reactive power (Q_T plus the reactive power absorbed by x_{Th}) must be supplied by the voltage source V_{Th}. On the other hand, if the Subsystem II is injecting reactive power into busbar L, Q_T is negative. In this case, the fictitious voltage source V_{Th} could even be receiving reactive power from Subsystem II, depending on how much reactive power is absorbed by the impedance of transference. The vector diagram of voltages and current for an inductive load connected to busbar L (in figure 2.1a) is shown in figure 2.1b.

Following the load-generator convention, if the transmission angle δ is positive for $\dot{V}_{Th}$ leading $\dot{V}_L$, the active power is negative when it leaves the sending-end busbar and positive when it reaches the receiving-end busbar. For a pure inductance such as x_{Th}, the sum of the supplied and absorbed active powers must be equal to zero.

Equation 2.1, derived from trigonometric relations taken from the vectors in figure 2.1b, relates the power P_T to the transmission angle δ ($P_T = f(\delta)$). Similarly, equation 2.2 relates the receiving-end voltage V_L to the transmission angle δ ($V_L = f(\delta)$).

$$P_T = \frac{V_{Th}{}^2 \sin(\delta)\cos(\delta+\phi)}{x_{Th}\cos(\phi)} \qquad 2.1$$

$$V_L = V_{Th}\,\frac{\cos(\delta+\phi)}{\cos(\phi)} \qquad 2.2$$

where: δ : transmission angle between $\dot{V}_{Th}$ and $\dot{V}_L$

ϕ : load angle $\left(\phi = tg^{-1}\left(Q_T/P_T\right)\right)$

V_{Th} : rms value of $\dot{V}_{Th}$

V_L : rms value of $\dot{V}_L$

Varying the transmission angle δ in both equations 2.1 and 2.2 and plotting P_T against V_L, the resulting curve is called ***power transfer characteristic*** or ***P-V characteristic*** of the ac system, presented in figure 2.1a.

The power transfer characteristic (or, in this case, the relation between P_T and V_L) is affected by changes either in the sending-end voltage magnitude or in the impedance of transference between sending and receiving ends, or even in the transfer power factor.

According to equation 2.1, the maximum value of power that can be transferred from the power source to the load is limited by the three parameters mentioned before. This power transfer limit has been defined as ***maximum transmissible power*** [49] or ***maximum available power*** (MAP) [50]. The maximum transmissible power can be also defined as the steady-state limit for power transfer.

From equation 2.1, for a non-supported receiving-end voltage, the MAP occurs at $\delta=0.5(90°-\phi)$. For instance, if the Subsystem II is purely resistive ($\phi=0$), the MAP is reached at $\delta=45°$.

The effect of each one of the three parameters (impedance of transference, transfer power factor and sending-end voltage) on the power transfer characteristic is individually analysed in the Sections 2.2.1, 2.2.2 and 2.2.3, respectively.

2.2.1 The power transfer characteristic and the impedance of transference

The influence of the impedance of transference on the power transfer characteristic (P-V curve) is analysed by keeping the sending-end voltage and the transfer power factor unchanged. Supposing that the impedance of the load in Subsystem II varies from infinite (open-circuit) to zero (short-circuit), the voltage V_L varies from V_{Th} to zero. From equation 2.2, this would correspond to a transmission angle δ varying from $(\delta+\phi)=0$ to $(\delta+\phi)=90°$. The power transfer characteristic for the system shown in figure 2.1a is achieved by varying δ simultaneously in both equations 2.1 and 2.2 and plotting the power P_T against V_L.

In example 2.1, the effect of three distinct values for impedance of transference x_{Th} on the power transfer characteristic is presented.

Example 2.1

To simplify the analysis, the Subsystem II (in figure 2.1a) comprises a purely resistive load. There is only the active power P_T; the reactive power Q_T is zero (unit power factor). Equations 2.1 and 2.2 become:

$$P_T = \frac{V_{Th}{}^2 \sin(2\delta)}{2x_{Th}} \qquad 2.3$$

$$V_L = V_{Th} \cos(\delta) \qquad 2.4$$

Using per unit based on the rated rms voltage at busbar L, three values for x_{Th} are considered: x_{Th1}=0.7 pu, x_{Th2}=0.5 pu and x_{Th3}=0.3 pu. A sending-end voltage V_{Th}=1.12 pu is chosen to result in V_L equal to 1.0 pu for P_T equal to 1.0 pu and x_{Th2} equal to 0.5 pu. In figure 2.2, each curve represents the power transfer characteristic for each one of the three values of x_{Th} replaced into equation 2.3. Curve 1 is the P-V characteristic for x_{Th1}, curve 2 for x_{Th2}, and curve 3 for x_{Th3}.

In figure 2.2, the maximum available power for curve 1 is P_{MAP1}= 0.93; for curve 2 is P_{MAP2}= 1.25 pu and for curve 3 is P_{MAP3}= 2.08 pu. The three MAPs occur at the same voltage V_{SM}=0.79 pu. If the power P_T varies without changing the power factor (in this example, unity power factor), the MAP is always reached at the same V_{SM}. Changes in x_{Th} *do not modify the voltage* at maximum power transfer, although the *MAP itself is modified*. Analysing the P-V characteristics in figure 2.2, for a given power demand and at a specified power factor (unity, in this example), the larger the impedance of transference x_{Th}, the larger the reduction of voltage magnitude at the receiving end.

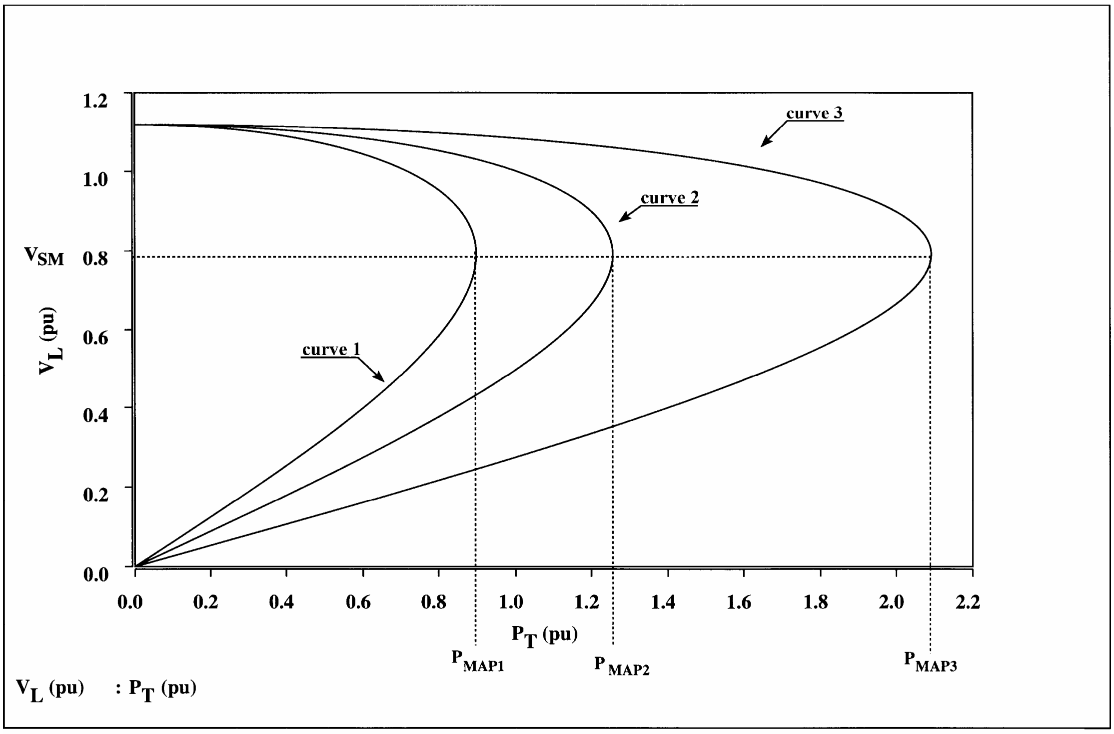


Figure 2.2 P-V characteristic for distinct values of x_{Th}

For resistive and inductive loads connected to busbar L (see figure 2.1a), the magnitude of the receiving-end voltage is very sensitive to changes in x_{Th}. Considering the power flow equal to 0.85 pu, the voltage V_L for x_{Th1} (curve 1 in figure 2.2) is equal to 0.91 pu. For the same power and x_{Th2}, V_L is equal to 1.04 pu, and for x_{Th3}, V_L is equal to 1.09 pu.

For capacitive loads, however, the effect of x_{Th} in the end voltage magnitude is far more modest. Considering, for instance, the power factor as equal to 0.98 (capacitive) and the same power flow as equal to 0.85 pu, the voltage V_L is equal to 1.09 pu for x_{Th1}. For x_{Th2}, V_L is equal to 1.13 pu; for x_{Th3}, V_L is equal to 1.14 pu. From these results, the effect of x_{Th} on the voltage magnitude of the receiving-end is primarily a function of the power factor at that end. Since typical power systems operate with high power factor, the influence of x_{Th} on the magnitude of the receiving-end voltage can be considered as a secondary effect.

Changes in x_{Th} affect mainly the transmission angle. If the active power P_T changes, the voltage V_{XTh} produced by the line current $\dot{I}$ crossing x_{Th} also changes. P_T is related to the component of current $\dot{I}$ in phase with $\dot{V}_L$ (active component). Since x_{Th}, in the present discussion, is assumed to be purely inductive, V_{XTh} is orthogonal to the in-phase current component (see figure 2.1b). Therefore, for typical power systems, V_{XTh} is quasi-orthogonal to $\dot{V}_L$. Change in x_{Th} modifies Vx_{Th} and, consequently, the transmission angle δ between $\dot{V}_{Th}$ and $\dot{V}_L$ is altered. The reduction of x_{Th}, for example, results in the reduction of the voltage across it, for a given power flow. As the transmission angle is directly related to the voltage across x_{Th} (voltage V_{XTh}, in figure 2.1b), the transmission angle is also reduced.

To illustrate the effect of x_{Th} into the transmission angle δ, the following example is discussed: from equation 2.4 (unity power factor), for a power flow equal to 0.85 pu, V_{Th} equal to 1.12 pu and x_{Th1}, the transmission angle δ is equal to 35.78°. Considering the same P_T=0.85 pu and V_{Th}=1.12 pu but with x_{Th2}, δ is equal to 21.33°; for x_{Th3}, δ is equal to 12.01°. The figures indicate a strong influence of the impedance of transference x_{Th} into the transmission angle δ.

Using equation 2.1 for the previous example, when the transfer power factor is equal to 0.98 capacitive, the transmission angle δ for x_{Th1} is 29.03°. For x_{Th2}, δ is 19.54°, and for x_{Th3}, δ is 12.03°. Comparing the transmission angles for each one of the three impedance of transference x_{Th} for both unity and capacitive transfer power factors, the influence of the transfer power factor on the transmission angle is small. Regardless the transfer power factor (for typical values), the impedance of transference x_{Th} remains as the main variable that affects the transmission angle δ.

The stability of the power system is related to the transmission angle δ. When the power is transferred at small transmission angle, the operating point of the ac is distant from the steady-state limit, increasing the stability margin [49][51]. Both the steady-state limit for power transfer and stability margin are defined in Section 2.4.1.2.

If the power demand at busbar L increases, a compensation of x_{Th} might be required to reduce the transmission angle and move the operating point away from the steady-state stability limit, increasing the stability margin.

The transfer power factor also affects the P-V characteristic. The relation between transferred power factor and power transfer characteristic is discussed in Section 2.2.2.

2.2.2 The power transfer characteristic and the transfer power factor (pf_T)

Equations 2.1 and 2.2 relate the transferred power P_T and the voltage V_L to the transfer power factor ($\cos\phi$). If P_T and V_L are individually affected by $\cos\phi$, so is the relation between them, which is represented by the power transfer characteristic. If the transfer power factor is capacitive, indicating an injection of reactive power into the receiving end, reactive losses in x_{Th} are partially (or totally, depending on the amount of injected reactive power) compensated.

The effect of $\cos\phi$ on the curve P-V is discussed in example 2.2. The same simplified system shown in figure 2.1a is adopted for the analysis.

Example 2.2

Considering the same figure 2.1a as in the example 2.1, the voltage V_{Th} is equal to 1.12 pu and x_{Th} is equal to 0.5 pu. To define the power transfer characteristic, as in Section 2.2.1, the load in Subsystem II varies from infinite to zero whilst V_{Th} and x_{Th} remain constant. However, three loads with distinct power factors are considered.

The P-V characteristics, presented in figure 2.3, for three distinct transfer power factors. In curve 1, the transfer power factor is pf_{T1}=0.98 inductive (lagging); in curve 2 is pf_{T2}=1.0 (unity transfer power factor) and in curve 3 is pf_{T3}=0.98 capacitive (leading). The transfer power factors pf_{T1}, pf_{T2} and pf_{T3} are kept constant during the variation of load for each one of the three P-V characteristics plotted in figure 2.3. The transfer power factor clearly affects the power transfer characteristic.

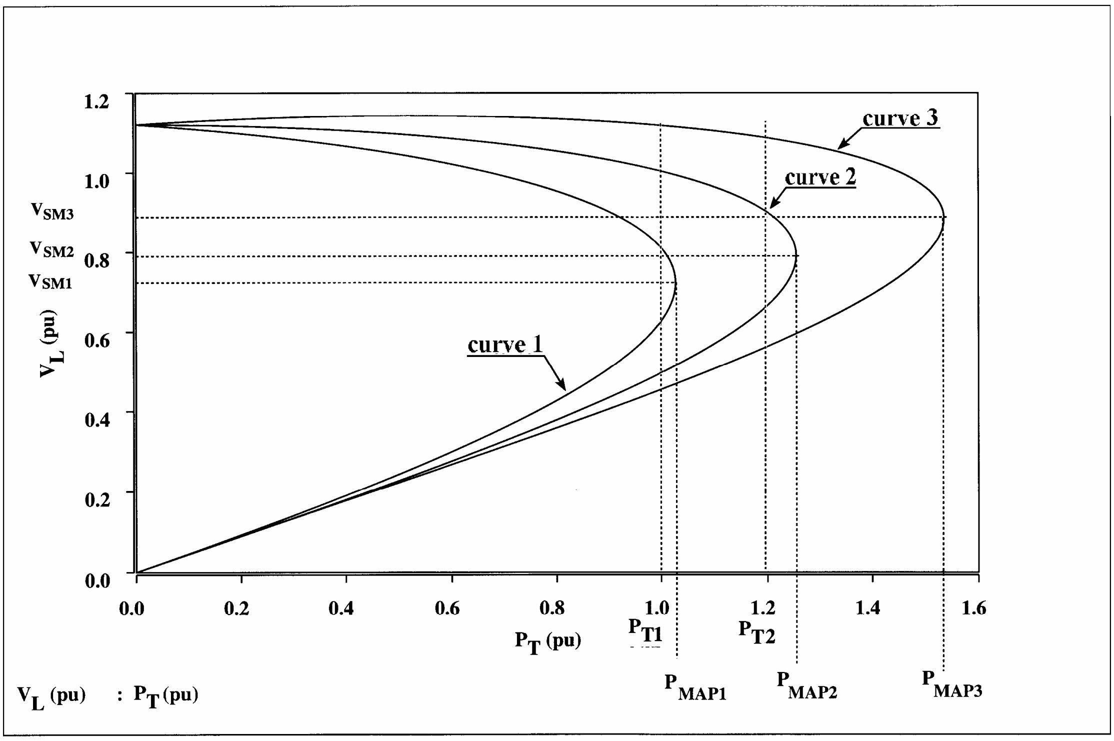


Figure 2.3 Power transfer characteristic for distinct transfer power factor

From figure 2.3, when the power P_{T1} equals 1.0 pu, the voltage V_L is equal to 0.81 pu for pf_{T1}, in curve 1. For pf_{T2} (curve 2, unity transfer power factor) and the same P_{T1}, the voltage is equal to 1.0 pu. For pf_{T3} (curve 3), the voltage is equal to 1.12 pu. Therefore, the more capacitive the transfer power factor the higher the voltage V_L, for a given power P_T.

Considering a second transferred power P_{T2} equal to 1.19 pu, it is not possible to transfer such power with pf_{T1}, since the maximum available power in curve 1 is smaller than 1.19 pu. For pf_{T2}, the voltage V_L is equal to 0.9 pu, which is 10 % smaller than the end voltage for power equal to 1.0 pu (P_{T1}). For pf_{T3}, V_L is equal to 1.09 pu. Moreover, for the leading power factor pf_{T3} (curve 3), it is possible to transfer up to 1.48 pu of power with a receiving-end voltage equal or higher than 0.9 pu.

The two cases discussed above are summarised in table 2.1.

Table 2.1 Effect of the transfer power factor on receiving-end voltage for two distinct values of power

cases (n)	P_{Tn}	V_{L1} (pf_{T1})	V_{L2} (pf_{T2})	V_{L3} (pf_{T3})
1	1.0	0.81	1.00	1.09
2	1.19	---	0.9	1.12

Growth of transferred power does not necessarily results in an intense reduction of the receiving-end voltage. For a leading power factor, for instance, the voltage can be sharply reduced only in the vicinity of the MAP.

Referring to figure 2.3, if the transfer power factor is corrected, the maximum available power can be significantly improved. In addition, by changing transfer power factor, the maximum power can be transferred at an acceptable voltage drop at the receiving end. Although the maximum power transfer is hardly considered under steady-state operations, it might be a temporary solution following a major disturbance in the ac system (transient condition).

From figure 2.3, the maximum available power P_{MAP1} is equal to 1.03 for pf_{T1} (curve 1), with the busbar voltage V_{LM1}= 0.72 pu. P_{MAP2} is equal to 1.25 for pf_{T2} (curve 2), with V_{LM2}= 0.79 pu. P_{MAP3} is equal to 1.53 for pf_{T3} (curve 3), with V_{LM3}= 0.88 pu. These values are arranged in table 2.2.

Table 2.2 Maximum available power and respective receiving-end voltages

pf_T	V_{LM}	P_{MAP}
0.98	0.72	1.03
1.00	0.79	1.25
-0.98	0.88	1.53

Correction of the transfer power factor can be achieved either by changing the power factor of the load or by injecting reactive power (reactive shunt compensation) into the busbar. Reactive shunt compensation is the usual solution to correct the transfer power factor.

The transfer power factor is an important aspect to be considered in the power transfer characteristic of the ac system. Power factor correction can affect both the transmission angle and the voltage magnitude. However, it is normally associated to shunt compensation, which is mainly used to control voltage magnitude. Aspects of shunt compensation are discussed in Section 2.3.1.

The magnitude of the sending-end voltage, the third parameter that modifies the power transfer characteristic, is discussed in Section 2.2.3.

2.2.3 The power transfer characteristic and the sending-end voltage magnitude

According to equations 2.1 and 2.2, the power transfer characteristic is affected by the magnitude of the sending-end voltage (V_{Th}).

For a given power P_T and impedance of transference x_{Th}, the sending-end voltage can be controlled to accomplish a specified receiving-end voltage. Handling equations 2.2 and 2.1, the transmission angle δ is calculated as follows:

$$\delta = tg^{-1}\left(\frac{P_T x_{Th}}{V_L^{\;2} + x_{Th} Q_T}\right) \qquad 2.5$$

The sending-end voltage is calculated by substituting the angle δ, in equation 2.5, into equation 2.2. In example 2.3, the influence of sending-end voltage on the P-V characteristic is demonstrated.

Example 2.3

In this example, the reactance x_{Th} is equal to 0.5 pu and the transfer power factor is equal to the unity ($\cos\phi=1$). For unity power factor, equation 2.5 is simplified as follows:

$$\delta = tg^{-1}\left(\frac{P_T x_{Th}}{V_L^{\;2}}\right) \qquad 2.6$$

In figure 2.4, the P-V characteristic for the simplified system given in figure 2.1a is presented, for two distinct values of the sending-end voltage V_{Th} (V_{Th1} and V_{Th2}). The Subsystem I is required to provide active power to the load connected to busbar L with a receiving-end voltage V_L equal to 1.0 pu.

In curve 1, the active power P_{T1} at the receiving-end is equal to 1.0 pu. The sending-end voltage V_{Th1} must be equal to 1.12 pu to result in a receiving-end voltage V_L equal to 1.0 pu.

In curve 2, the active power increases to P_{T2}=1.2 pu and must also be supplied at V_L=1.0 pu. To satisfy such condition, the sending-end voltage is increased to V_{Th2}=1.166 pu.

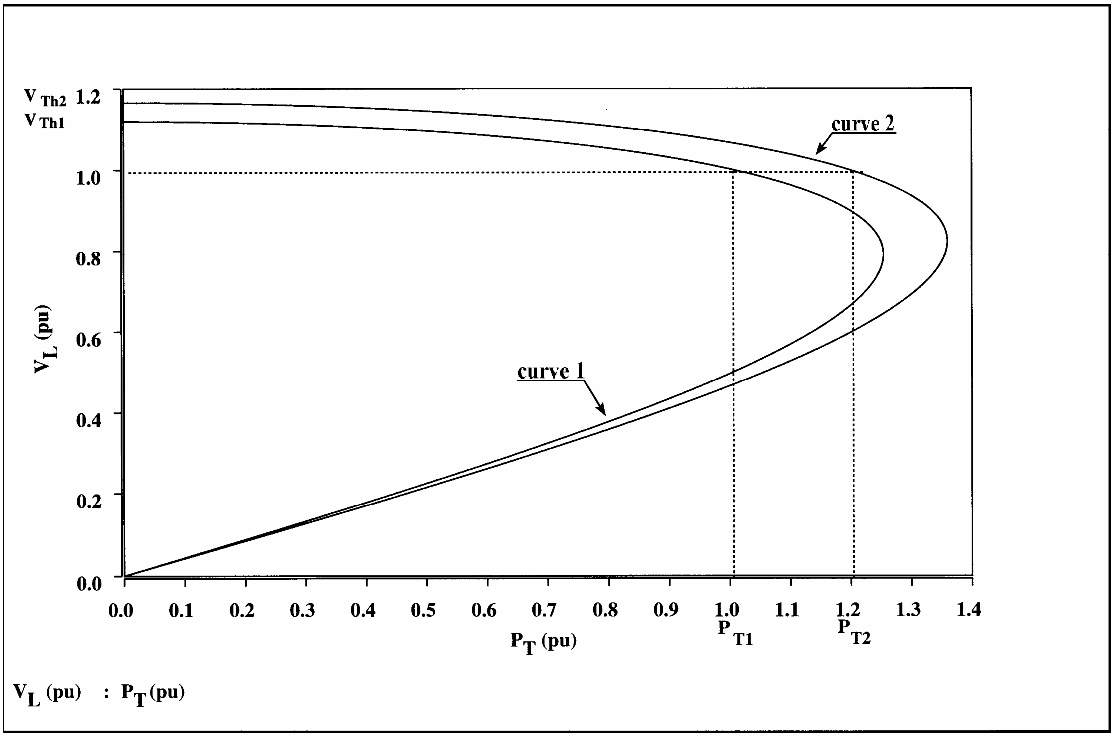


Figure 2.4 Power characteristic for two distinct sending-end voltages

Based on the curves illustrated in figure 2.4, the influence of the magnitude of the sending-end voltage on the power transfer characteristic is promptly demonstrated, which implies that the control of the receiving-end voltage V_L can be achieved by controlling the sending-end voltage V_{Th}.

The curves in figure 2.4 also suggest that the magnitude of the sending-end voltage does not affect substantially the maximum available power, when compared to the figures presented in Sections 2.2.1 and 2.2.2, at least for typical values of sending-end voltages.

The results discussed in this section allows to infer that the control of one or more variables involved in a power transfer, namely sending-end voltage, impedance of transference and transfer power factor, modifies the power transfer characteristic and, as a consequence, the operating condition of a power transmission system. Using reactive compensation, the former two variables can be controlled. This is discussed in Sections from 2.3 to 2.3.2.

2.3 Power Transfer and Reactive Compensation

According to previous discussions presented in Sections 2.2.1 and 2.2.2, changes in impedance of transference and correction of transfer power factor effectively affect the power transfer characteristic of a transmission system. If those parameters are properly controlled, the power transfer characteristic can be modified to attend to a required operating condition.

The control of the power transfer characteristic has traditionally been performed by reactive compensation [49]. The reactive compensator can be either shunt or series-connected (or even a combination of both connections) to the power systems. There are many designs of reactive compensators, from the simple capacitor bank mechanically switched to the modern thyristor-based controllers, such as the static var compensator (SVC), the advanced SVC (ASVC) or the thyristor-controlled series compensator (TCSC) [22][52][53][54][55][56]. Details on each one of the possible designs for the reactive compensators are beyond the scope of this research. However, the influence of shunt and series reactive compensations on the power transfer characteristic is discussed in Sections 2.3.1 and 2.3.2.

2.3.1 Power transfer and shunt compensation

In the past, reactive shunt compensation used to be performed by merely switching on and off capacitor and reactor banks into the ac system or by connecting over-excited synchronous machine to the system as synchronous condenser. The advance of power electronics in the last decades has contributed to the design of compact, highly controllable shunt compensators.

Reactive shunt compensators supply part of reactive power required by the ac system and they are normally used to maintain the local voltage at a specified (ordained) value (or range of values). Figure 2.5 shows a shunt compensator connected to the ac system presented in figure 2.1a.

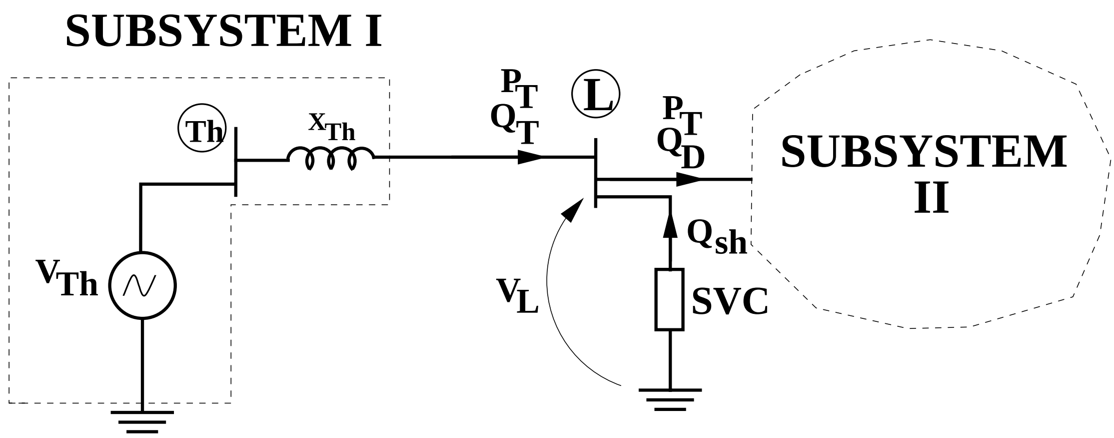


Figure 2.5 Shunt compensator connected to a simplified ac system

The shunt compensator illustrated in figure 2.5 could be a SVC [49] or any compensator that provides both capacitive and inductive compensations. The SVC injects reactive power Q_{sh} into busbar L. The total reactive power reaching the busbar L, Q_T, is now the sum of the reactive power required by the Subsystem II, Q_D, plus the reactive power injected by the reactive compensator, Q_{sh}. The injected reactive power Q_{sh} can be either leading (capacitive) or lagging (inductive), depending on the requirements of the ac system. Considering an ideal operation of the SVC, for instance, the voltage V_L would be kept at a controlled value V_{Lord}.

In equation 2.2, when the voltage V_{Lord} is equal to V_{Th}, the transmission angle δ is related to the load angle ϕ as follows:

$$\cos(\delta+\phi)=\cos(\phi) \qquad 2.7$$

To satisfy equation 2.7, the relation between the angles δ and ϕ must be:

$$\phi = -\frac{\delta}{2} \qquad 2.8$$

Therefore, for changes in power flow, which would result in changes in transmission angle δ, the load angle ϕ must be continuously modified to satisfy equation 2.8. Substituting equation 2.8 into equation 2.1:

$$P_T = \frac{V_{Th}^2 \sin(\delta)}{x_{Th}} \qquad 2.9$$

At first glance, equation 2.9 does not seem directly related to the transfer power factor ($\cos\phi$). However, after analysing equations 2.2 and 2.7, it is verified that V_L is kept to an ordained value for changes in power flow only if the load angle ϕ is continually modified. In other words, it is necessary to control the transfer power factor to keep V_L at an ordained value. The shunt compensator performs such control.

A comparison between equations 2.3 and 2.9 can be established. Equation 2.3 relates P_T to δ for resistive load connected to a busbar *without voltage support* whilst equation 2.9 relates P_T to δ in a busbar *with voltage support*, regardless of the load. The maximum power transfer calculated using equation 2.9 is exactly twice the maximum power transfer calculated using equation 2.3. Furthermore, the maximum power calculated in equation 2.9 occurs at a given transmission angle δ which is twice the angle δ calculated in equation 2.3 for the maximum power.

Examining equation 2.1 (a general condition) and equation 2.9, one can deduce that a given active power is transferred at smaller transmission angle δ when the receiving-end voltage is supported by reactive compensation. Additionally, the maximum available power is greatly improved.

The conditions described on equations 2.1 and 2.2 have no practical meaning when dealing with real transmission systems because only a narrow margin of variation of voltage magnitude at the system busbars is allowed (usually 5% below or above the nominal voltage). However, the comparison between equations 2.1 and 2.9 is justified since it illustrates the importance of supporting voltage for power transfer.

For long transmission lines, the midpoint shunt compensation is an alternative to improve power transfer. Midpoint shunt compensation is presented in Section 2.3.1.1.

2.3.1.1 Midpoint shunt compensation

A feasible procedure to improve power transfer through long transmission lines whose end voltages are already controlled is to place the shunt compensation into the middle of the line, as illustrated in figure 2.6a [49].

To exemplify the mid-point shunt compensation, a long transmission line connecting two identical subsystems is shown in figure 2.6a. The end voltages are kept constant and represented by ideal voltage sources (E_s and E_r). A shunt reactive compensator is placed into the middle of the transmission line (point m), splitting the line series reactance into two halves (equal to $x_l/2$). Each half of the transmission line is represented by a π-equivalent circuit. The voltage at the midpoint is called V_m and the shunt compensator injects the reactive current I_{sh} into the point m.

The current I_{sh} can be either capacitive or inductive, depending on the degree of compensation required at the midpoint. The relation between the transmission line shunt capacitance and the shunt compensator reactance can be expressed as follows:

$$k_m = \frac{B_\gamma}{\frac{1}{2}B_c} \qquad 2.10$$

where: B_γ : compensating susceptance

B_C : total transmission line susceptance

Source: T. J. Miller

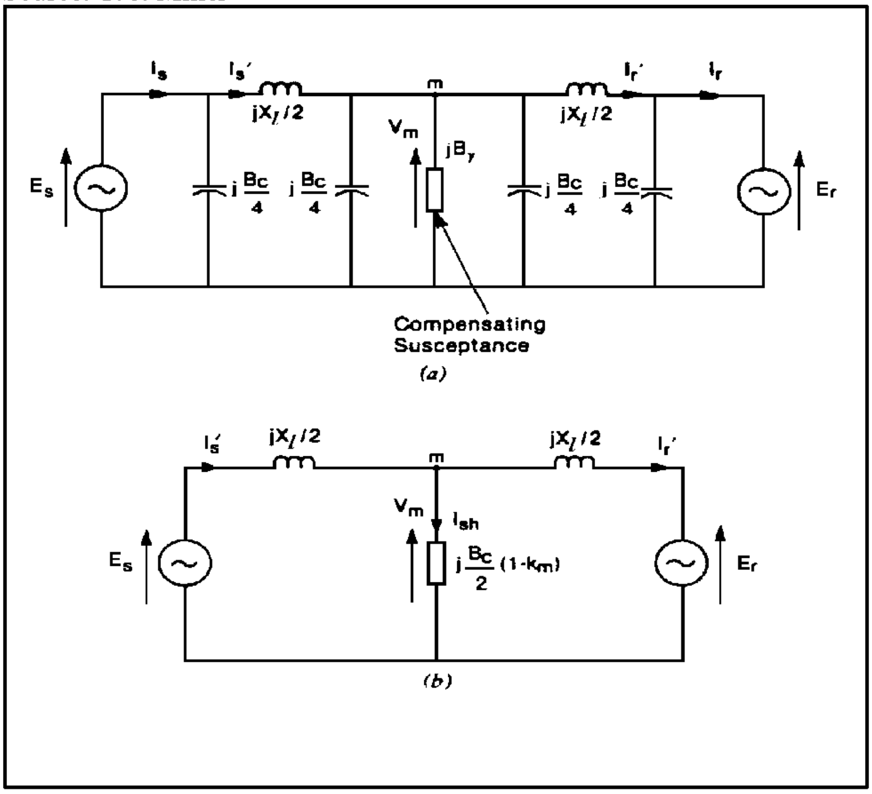


Figure 2.6 Midpoint shunt compensation for a long transmission line

Considering the end voltages constant, the circuit presented in figure 2.6a can be simplified, resulting in the circuit shown in figure 2.6b. The central susceptance, in figure 2.6b is a combination of the compensator susceptance (B_{γ}) plus the transmission line susceptance (B_C) and can be expressed as:

$$B_{central} = \frac{B_c}{2}\left(1 + k_m\right) \qquad 2.11$$

If k_m is negative, the shunt compensator works as an inductance and its susceptance lessens the transmission line susceptance.

On the other hand, if k_m is positive, the shunt compensator acts as a capacitance, increasing B_C. If $B_\gamma = -B_C/2$ (inductive), k_m is equal to −1 and the central susceptance is equal to zero.

Applying the Δ–Y transformation to the circuit described in figure 2.6b, and considering E_s and E_r constant (together they must absorb half of the reactive power provided by the transmission line capacitance), the impedance of transference between E_s and E_r can be expressed as:

$$x_{Th} = x_l(1-s) \qquad 2.12$$

where:

$$s = \frac{x_l}{2}\frac{B_c}{4}(1-k_m) \qquad 2.13$$

Considering $E_s = E_r = V_{Th}$ and substituting equation 2.12 into 2.9, the power equation can be described as:

$$P_T = \frac{V_{Th}^2 \sin(\delta)}{x_l(1-s)} \qquad 2.14$$

Applying the midpoint compensation to the system shown in figure 2.1a, dividing x_{Th} into two equal halves and keeping the busbar voltage V_L equal to V_{Th}, equation 2.14 can be re-written as:

$$P_T = \frac{2V_{Th}^2 \sin(\delta/2)}{x_{Th}} \qquad 2.15$$

Comparing now equation 2.9 with equation 2.15, the maximum power transfer with shunt compensation in the middle of the transmission line impedance (equation 2.15) is twice the power transfer without midpoint compensation (equation 2.9).

The maximum power calculated using equation 2.15 occurs for a transmission angle δ which is twice the δ for maximum power when no shunt compensation is provided in the midpoint (equation 2.9). The compensation procedure by sectioning the transmission line into two halves has been proposed to improve maximum transmissible power in long transmission lines [46].

In figure 2.7, the power-angle (P–δ) characteristics calculated using equations 2.3, 2.9 and 2.15 are compared. Curve 1, which is computed using equation 2.3, represents the P–δ characteristic of the simplified system presented in figure 2.1a, without support in the voltage V_L. Curve 2 represents the P–δ characteristic of the system shown in figure 2.5, with V_L constant and equal to V_{Th}, and it is calculated using equation 2.9. Curve 3 illustrates the P–δ characteristic for supported voltage at the midpoint of the impedance x_{Th}, according to figure 2.6a and it is calculated using equation 2.15.

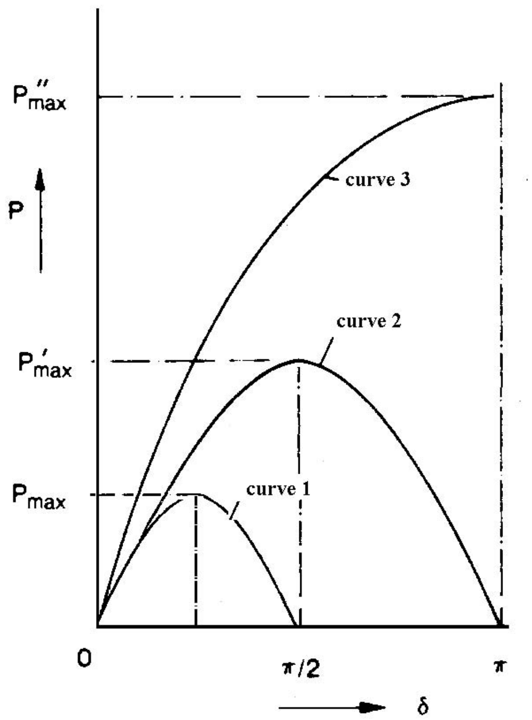


Figure 2.7 The effect of shunt compensation on power-angle characteristic

Despite considering the end voltages as ideally controlled and transmission systems without losses, the results presented in the present section illustrate how shunt compensation modifies the power transfer characteristic. It is possible to improve the power transfer at small transmission angle by using shunt compensation. This achievement has significant impact on the power system stability performance.

In Section 2.3.2, the influence of series compensation on power transfer characteristic is presented.

2.3.2 Power transfer and series compensation

Capacitor (or capacitor bank) inserted in series with the transmission line has traditionally performed series compensation. However, problems such as sub-synchronous resonance [50] (which originates from the combination between compensating capacitors and generator inductances), have justified the search for new concepts in series compensation. The Thyristor-Controlled Series Compensator (TCSC), which uses Thyristor-Switched Capacitor (TSC) in parallel with Thyristor-Controlled Reactor (TCR), is one of the new controllers used as series compensators which are sub-synchronous resonance free [25][26][42][57]. Furthermore, controllable series compensation has been claimed to improve the power oscillation damping and to re-direct the power flow, avoiding undesirable power loops [58].

To exemplify the effect of series compensation on the power transfer characteristic, a generic reactive compensator x_{se} is inserted in series with x_{Th} in the simplified system represented in figure 2.1a. The modified system is illustrated in figure 2.8.

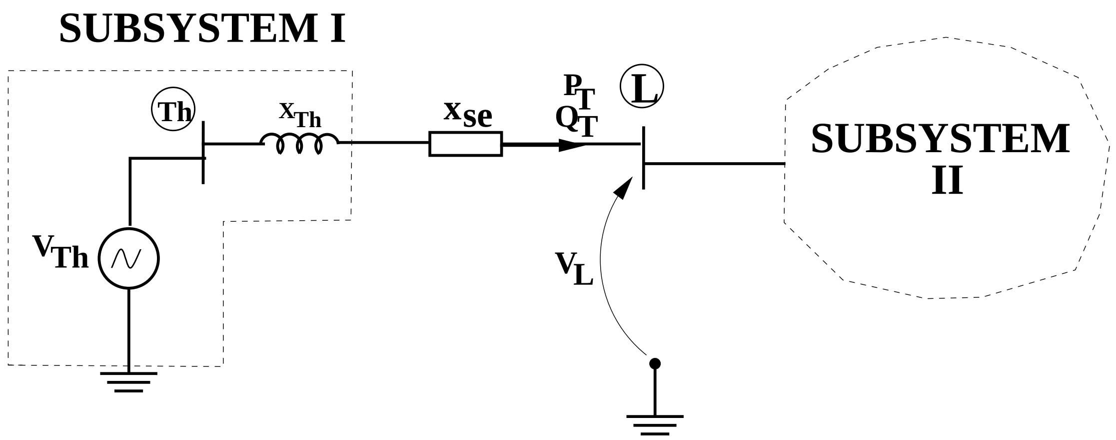


Figure 2.8 Series compensation in a simplified system

As the reactance x_{se} is inherently capacitive, it is subtracted from x_{Th}, which is essentially inductive. Equation 2.1 is then modified, resulting in equation 2.16 as follows:

$$P_T = \frac{V_{Th}^{\;2}\sin(\delta)\cos(\delta+\phi)}{x_{Th}(1-k_{se})\cos(\phi)} \qquad 2.16$$

where $k_{se} = \dfrac{x_{se}}{x_{Th}}$: compensating index

The compensating index k_{se} modifies the impedance between busbar Th and busbar L, in figure 2.8. For a constant $V_{L,}$ equation 2.16 becomes:

$$P_T = \frac{V_{Th} V_L \sin(\delta)}{x_{Th}(1-k_{se})} \qquad 2.17$$

The series compensation reduces the impedance of transference between two busbars. Therefore, for a given power transfer, the voltage across the impedance of transference also decreases, resulting in a smaller transmission angle between $\dot{V}_{Th}$ and $\dot{V}_L$.

On the other hand, when using reactive series compensation, the power transfer can be increased at a reduced transmission angle, keeping the operating point distant from the maximum power transfer. Power transfer at a small transmission angle is especially important when the transmission line is an inter-tie, connecting two major subsystems. Large transmission angles can result in loss of synchronism between interconnected machines when the ac system is subject to disturbances. Several papers have reported improvement on power system stability when series reactive compensation is used [22][28][31][59].

Curve 1, in figure 2.9, which is drawn from values calculated using equation 2.17, represents the effect of series compensation on the power-angle characteristic. Curve 2 represents the power-angle characteristic without series compensation $\left(k_{se} = 0\right)$. In both cases, V_L is considered constant and equal to V_{Th}.

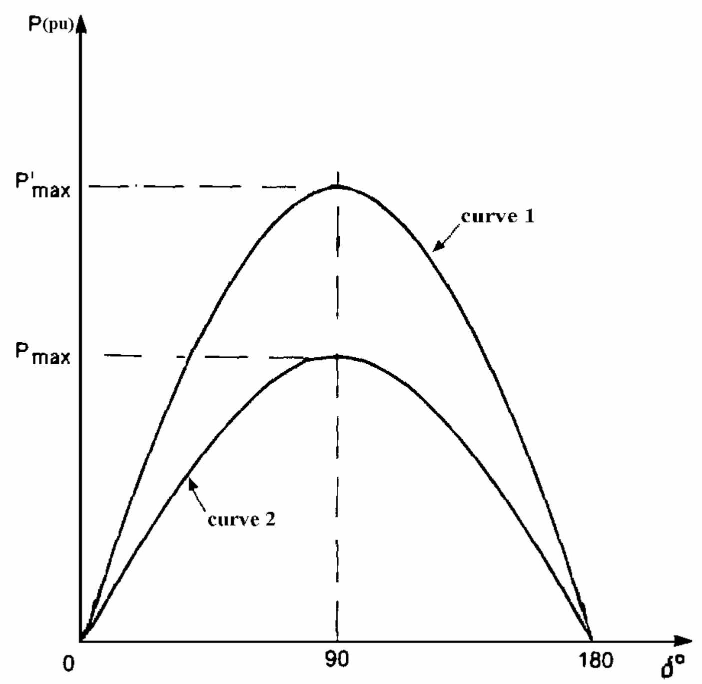


Figure 2.9 The effect of series compensation on power-angle characteristic

Based on figure 2.9, series compensation increases the maximum power, from P_{max} to P'_{max}. However, the transmission angle for maximum power is unchanged. This is readily verified by examining equations 2.16 and 2.17, since series compensation affects only the reactance, without interfering in the transmission angle.

The analysis conducted in Sections 2.2 and 2.3 consider generic impedance of transference. However, the transmission line is the electrical component responsible for transferring power in electrical power systems. The behaviour of the transmission line as a power transfer-limiting factor, regarding both its electric parameters and class of voltage, is discussed in Section 2.4.

2.4 Transmission Line and Power Transfer

In electrical power systems, the transmission lines are the component responsible for transferring power from a source to a load or, broadly speaking, from one point to another in the system. Paradoxically, the transmission line also limits power transfer, due to its own capability characteristics. St. Clair first introduced practical concepts on the capability of transmission lines in the fifties [60]. The main factors limiting the transmission line capability are the system voltage level, the thermal rating (line current) steady-state stability limit and the exchange of reactive power among transmission line reactances. Resistive losses and temperature are also limiting factors, but of limited significance. Voltage level and thermal rating are based on constructive aspects of the transmission line, whilst steady-state stability limit and reactive exchange depend on the dynamic comportment of the power system.

The system voltage level is usually defined in the planning stage, according to the particularities of the system regarding power generation and distribution, such as available power sources, expected power demand, electrical distance between generation and load, among others.

Thermal rating is usually a limiting factor on the capability of relatively short transmission line, whose length is shorter than 80 km [50 miles] at fundamental frequency of 60 Hz [60]. Thermal rating is only defined considering constructive aspects of a transmission line. Then, it becomes more an economical rather than an operational problem. This is especially true for a voltage level of 138 kV or below.

At levels such as extra high voltage (EHV, from 230 kV up to 765 kV) and ultra high voltage (UHV, above 756 kV), the corona discharges and the field effects will require transmission line designs which result in a high thermal capability for a transmission line [61].

As the length of transmission line increases, the other two main factors, namely steady-state stability limit and reactive exchange, become the major limiting factors in the transmission line capability. The capability limits of the transmission line, regarding reactive exchange and steady-state stability, are discussed in Section 2.4.1.

2.4.1 Capability limits for transmission lines

The capability of the transmission line is usually expressed in terms of *surge impedance loading* (SIL). The surge impedance loading of a transmission line is defined as the power that flows in a lossless transmission line ending in a resistive load equal to the transmission line surge impedance. The surge impedance (Z_C) for a lossless transmission line is given by:

$$Z_C = \sqrt{\frac{l}{c}} \qquad 2.18$$

where: l: transmission line inductance (per unit of length)

c: transmission line capacitance (per unit of length)

The surge impedance loading (SIL) is expressed as:

$$SIL = \frac{\left(V_{phase\text{-}phase}\right)^2}{Z_C} \qquad 2.19$$

where: $V_{phase\text{-}phase}$: rms phase-to-phase voltage at the transmission line end, in kV
Z_C: transmission line surge impedance, expressed in ohms or without unit

If Z_C is expressed in ohms, SIL is given in MW. If Z_C has no unit, SIL is given in kV^2.

If the transmission line ends at its surge impedance, the voltage at the sending end is equal to the voltage at the receiving end. In this case, a line charging (capacitance) effect offsets the reactive absorbed by the transmission line inductance, resulting in a net reactive power in the transmission line equal to zero. In this condition, the transmission line is called ***flat*** or ***infinite*** line.

Power transmission lines rarely end in surge impedance. If the resistive load connected to the transmission line end is larger than the surge impedance Z_C, there is a surplus of reactive power provided by the transmission line capacitance. This is often referred to as *light loading condition*. The voltage at the transmission line receiving-end is higher than the voltage at the sending end. Conversely, for an ending resistive load smaller than Z_C, referred to as *heavy loading condition*, reactive losses exceed the reactive power provided by the transmission line capacitance. In this case, the receiving-end voltage is smaller than the sending-end voltage. The transmission line may be subject to a variation between heavy and light loading due to dynamic changes in the load. The result is a voltage fluctuation at the receiving-end of the transmission line, associated to reactive exchange imposed by the transmission line.

2.4.1.1 Reactive Power and the Transmission Line

The reactive power absorbed by the transmission line inductance, Q_l, is a function of the current passing through the transmission line and it is expressed according to equation 2.20. On the other hand, the reactive power provided by the transmission line capacitance, Q_c, is a function of the rated voltage at the transmission line end and it is expressed by equation 2.21.

$$Q_l = I^2 x_l \tag{2.20}$$

$$Q_c = \frac{V^2}{x_c} \tag{2.21}$$

where: I: rms current through the transmission line

V: rms phase-to-phase voltage

x_l: transmission line inductance $\left(x_l = 2\pi f l l, f = \text{fund. freq.}, l = \text{length}\right)$

x_c: transmission line capacitance $\left(x_c = \frac{1}{2\pi f c} l, f = \text{fund. freq.}, l = \text{length}\right)$

Q_c can also be expressed as a function of the line-charging current (I_{chg}), according to equation 2.22. Line-charging current, defined in equation 2.23, is the current resultant from the charging and discharging of a transmission line due to alternating voltage [62]. I_{chg} flows even if the receiving end of the transmission line is an open-circuit. As the voltage varies along the transmission line, the rated voltage is normally used to calculate the line-charging reactive power.

$$Q_c = \sqrt{3} I_{chg} V \tag{2.22}$$

$$I_{chg} = \frac{V}{x_c \sqrt{3}} \tag{2.23}$$

2.4.1.2 Steady-state stability and power-angle (P-δ) characteristic

The steady-state stability is defined as the ability of the power system to sustain the transfer of power from one terminal to another for small changes in load [50][51][63]. The limit of steady-state stability was defined as "a condition of a linear system or one of its parameters which places the system on the verge of instability [51]."

The steady-state stability is associated with the voltage across the transmission line reactance. As the current crossing the transmission line increases, the voltage across the transmission line reactance also increases. The voltage across this reactance is related to the angle between the voltage vectors at the ends of transmission line. This angle was previously defined as transmission angle. The larger the voltage across the transmission line, the larger the transmission angle.

For a flat line (as defined in Section 2.4.1), the relation between the transmitted power P and the transmission angle δ is expressed in equation 2.24.

$$P = \frac{V^2 \sin(\delta)}{x_l} \qquad 2.24$$

The relation between the transferred power P and the transmission angle δ (known as P–δ characteristic) for a flat transmission line is shown in figure 2.10. From equation 2.24 and figure 2.10, the maximum power transfer occurs when the transmission angle is equal to 90°. This is also the steady-state stability limit for the condition described above.

The steady-state stability of the system is given in terms of stability margin. Stability margin is indicated in figure 2.10. In practical terms, stability margin consists of determining a reasonable amount of power to be transferred across the transmission line that would keep the system under a stable operating condition following a predictable contingency. This contingency could be switching operation, outage of load or generation, changes in power dispatches, etc.

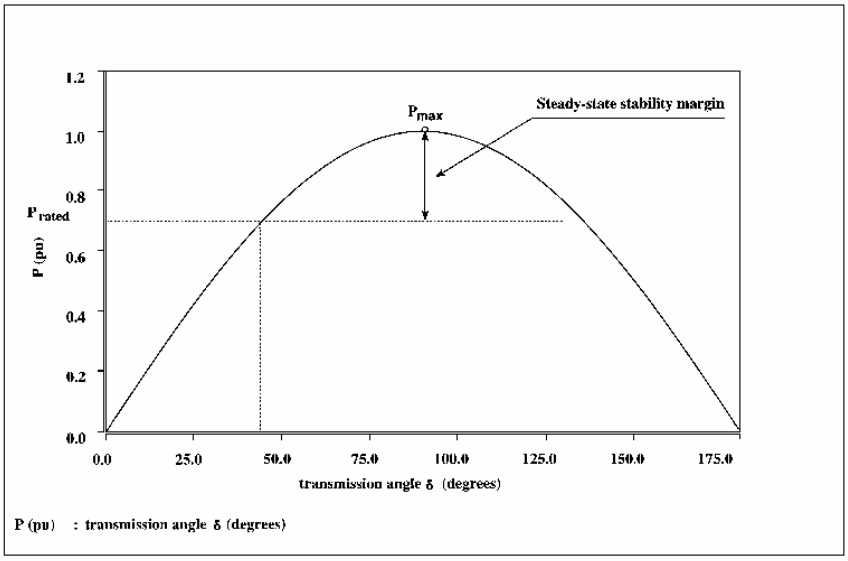


Figure 2.10 Power angle (P-δ) characteristic

Stability margin is defined according to equation 2.25:

$$\text{Stability Margin (\%)} = \frac{P_{max} - P_{rated}}{P_{max}} = 1 - \sin\delta \qquad 2.25$$

The steady-state stability margin, derived from the P-δ characteristic, is based on fixed voltage magnitude at the transmission line ends. This presumes that the reactive power absorbed by the transmission line is supplied by the ac system. Therefore, the steady-state stability is fundamentally influenced by the voltage across the transmission line series reactance, rather than by the net reactive power exchanged between the transmission line and the ac system.

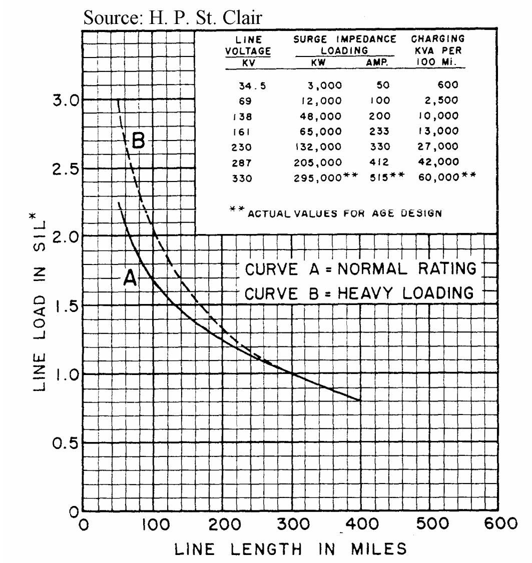
2.4.2 St. Clair capability curves

In the early fifties, H. P. St. Clair related the length with the capability of the transmission line by the curves known as St. Clair's capability curves [60]. St. Clair's capability curves, which were empirically derived for transmission lines up to 1000 km (600 miles), are shown in figure 2.11a. These curves were later analytically demonstrated [61]. The analytical curve, which is very similar to that shown in figure 2.11a, is shown in figure 2.11b.

The analytical demonstration [61] of the empirical St. Clair's capability curves clarifies the influence of each limiting factor on the transmission line capability. The analytical curve was created using two of the capability limiting factors previously discussed: voltage drop at the transmission line receiving end and steady-state stability margin.

In [61], the authors assume a maximum voltage drop at the receiving end equal to 5% and a stability margin equal to 35%. In figure 2.11b, curve 1 indicates the capability curve for a stability margin of 35%, considering a flat transmission line. Curve 2 shows the capability curve for a maximum voltage drop of 5%. Taking the more conservative value of both curves, the composed capability curve (in pu of SIL) matches the empirical curve presented in [60]. The region of voltage drop dominates the capability limits for transmission lines from 80 km (50 miles) to 320 km (200 miles). On the other hand, the region of stability margin limits the capability of transmission lines longer than 320 km. The limiting regions are identified in figure 2.11b.

Voltage drop and steady-state stability limits, as opposed to system voltage level and resistive losses, can be dynamically changed if the control of reactive power and transmission angle, respectively, can be established. In the Section 2.2, the relation between the system voltage vectors and the power transfer characteristic was demonstrated.



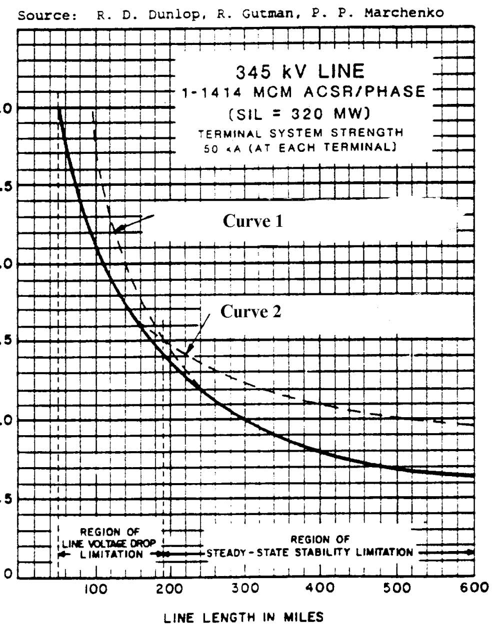
LINE VOLTAGE KV	SURGE IMPEDANCE LOADING KW	SURGE IMPEDANCE LOADING AMP.	CHARGING KVA PER 100 Mi.
34.5	3,000	50	600
69	12,000	100	2,500
138	48,000	200	10,000
161	65,000	233	13,000
230	132,000	330	27,000
287	205,000	412	42,000
330	295,000**	515**	60,000**

** ACTUAL VALUES FOR AGE DESIGN

A and B curves, normal and heavy loading basis for capability of transmission lines in terms of SIL

* Note: SIL = approximately 2.5 kv² (except 330-kv design = 2.7 kv²)

(a) an empirical approach



(b) an analytical approach

Figure 2.11 Transmission line capability curves

2.5 Conclusion

The analysis conducted in this Chapter 2 presented the limits of power transfer in typical electric power systems.

Power transfer (P-V) characteristics were plotted using simplified systems, for distinct conditions of load and generation. The effect of three system variables (namely the sending-end voltage magnitude, the impedance of transference between the sending and the receiving ends, and the transfer power factor) on the P-V characteristic was investigated. If the three system variables are efficiently controlled, the active power can be transferred according to a pre-defined operating condition.

The series and the shunt reactive compensations were proved to have a strong influence on the power transfer characteristic. Therefore, reactive power compensators can be used to control the operating condition in a power transfer operation.

Power transfer limits based on transmission line characteristics were introduced using the St. Clair's capability curves. Limiting capability factors such as voltage drop and steady-state stability were described. Although capability criteria have been associated to the transmission line parameters, the power transfer limitation would involve the total reactance between two busbars, including transformers, series and shunt system components, etc. Power controllers can be used to change the equivalent reactance between two busbars and to improve the power transfer capability.