

# Investigation of the dynamic characteristics of a smart tensegrity structure

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*Abstract: A experimental study of a tensegrity structure unit cell incorporating a SMA (shape memory alloy) element has been carried. A tensegrity system is composed by two types of elements, tensile parts and compressive parts, which promotes the structural stability of the system. In this study, the SMA element acts as tensile part that can be modified by changing the alloy temperature. The system has been modeled using the finite element method and the model was fitted based on the results of an experimental test. The results shows that the dynamic characteristics of a SMA tensegrity structure can be easily adjusted changing the tension on the cables.*

**Keywords:** Tensegrity, Vibration, Finite element method, SMA

## INTRODUCTION

A tensegrity system is composed by two types of elements, tensile parts and compressive parts, which promotes the structural stability of the system, in this case the tensile parts are cables, and the compressive parts are bars (Skelton, Oliveira de, 2009). An attractive characteristic of a tensegrity structure is the capability to be deployable, for this reason, is interesting for space applications like presented by Tibert (2002), due to the small volume that occupy in a possible transport outside of the earth.

There was a vast number of researches on tensegrity-based structures, like presented by Sultan (2009). The studies shown by Motro et al. (1987) presents numerical and dynamic experimental work using a tensegrity structure composed by three bars and 9 cables, with no control. Kebiche et al. (1999), showing that mechanical behaviour of self-stressed reticulated spatial systems is non-linear due to their flexibility, and also highlights that would be interesting to model the structure with different rigidities layers. Oppenheim, Williams (2001b,a) use simple elastic tensegrity structure and examined the dynamic behavior, shows that the tensegrity system can function as a counterpart to conventional trusses, even though they argue that, their system was ineffective at eliminating vibrations. Sultan et al. (2002) uses linear models which describes the approximate dynamics of tensegrity structures, and derives two classes of tensegrity structures showing that the modal dynamic range commonly increases with pretension.

Tensegrity systems can be also controlled to achieve some determined results like, Chan et al. (2004) presented a small scale active three bar structure, with local integral force feedback and acceleration feedback control, achieving effective damping for the next 2 resonant bending modes. Ali, Smith (2010) showed that natural frequencies can be shifted when the self-stress level in the tensegrity structure is modified, using active control. Amouri et al. (2013) uses robust active control algorithm ( $H_\infty$ ) in the vibration control of the tensegrity, and shows that provides a efficient performance on the attenuation of the modes, in one degree of freedom.

## ANALYTICAL FORMULATION

A linearized equation of motion to describe a dynamic model around an equilibrium configuration, is used to describe the dynamic behavior of the active tensegrity structure Ali, Smith (2010), as follow:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \quad (1)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$ , are mass, damping and stiffness matrices. The force ( $\mathbf{f}$ ) is the applied load vector, and  $\ddot{\mathbf{u}}$ ,  $\dot{\mathbf{u}}$  and  $\mathbf{u}$  are the vectors of nodal acceleration, velocity and displacement respectively. With the development of the tensegrity structure using finite element method, the parameters of mass, damping and stiffness are represented by matrices. The element mass matrix is represented by:

$$\mathbf{M}_e = \frac{\rho AL}{6} \begin{bmatrix} 2\mathbf{I}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & 2\mathbf{I}_3 \end{bmatrix} \quad (2)$$

Like shown by Ali, Smith (2010) and Guest (2006), the stiffness matrix  $\mathbf{K}$  can be decomposed into two other matrix,  $\mathbf{K}_E$  and  $\mathbf{K}_G$ , linear stiffness matrix and the geometrical stiffness matrix correspondingly,  $\mathbf{K} = \mathbf{K}_E + \mathbf{K}_G$  where:

$$\mathbf{K}_E = \frac{EA}{L} \begin{bmatrix} \mathbf{I}_0 & -\mathbf{I}_0 \\ -\mathbf{I}_0 & \mathbf{I}_0 \end{bmatrix} \quad \mathbf{K}_G = \frac{T}{L} \begin{bmatrix} \mathbf{I}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & \mathbf{I}_3 \end{bmatrix} \quad \mathbf{I}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The damping value is treated as equivalent Rayleigh Damping  $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ .

The structure used in this work is composed by four bars and 12 cables, and has 8 nodes localized at the joints, one of the cables is a shape memory alloy (SMA), and can be represented by the Fig. 1(a), and Fig. 1(b) presents the real model. In the illustration of Fig. 1(a), the thick black lines represent rigid bars, the cables are represented in thin blue lines and

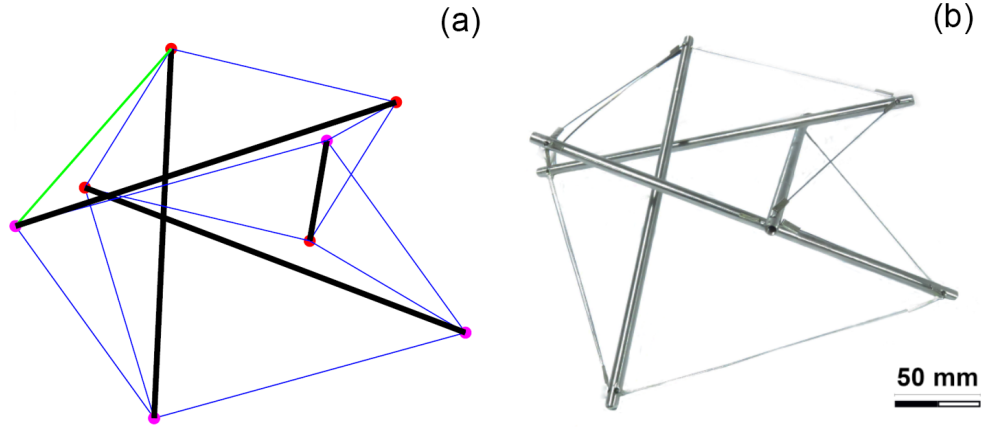


Figure 1 – Illustration of the tensegrity system, (a) model and (b) real system.

the the SMA cable in green color. In the analysis of this system, the red nodes where considered free while the pink nodes where fixed. The bars and the cables are made of stainless steel, and the SMA is made by nitinol. The relevant properties for this study using FEM, are, the cross section area ( $A$ ), the material density ( $\rho$ ), the material Young's modulus ( $E$ ), and tension of the cable( $T$ ). The bars properties are,  $E = 200$  [GPa],  $A = 2.8274 \times 10^{-5}$  [m<sup>2</sup>],  $\rho = 7.85 \times 10^3$  [g/cm<sup>3</sup>]. The cable properties are,  $E = 200$  [GPa],  $A = 3.3183 \times 10^{-7}$  [m<sup>2</sup>],  $\rho = 7.85 \times 10^3$  [g/cm<sup>3</sup>]. The SMA cable properties are,  $E = 40$  [GPa],  $A = 7.854 \times 10^{-7}$  [m<sup>2</sup>],  $\rho = 6.45 \times 10^3$  [g/cm<sup>3</sup>], when heated  $E = 70$  [GPa].

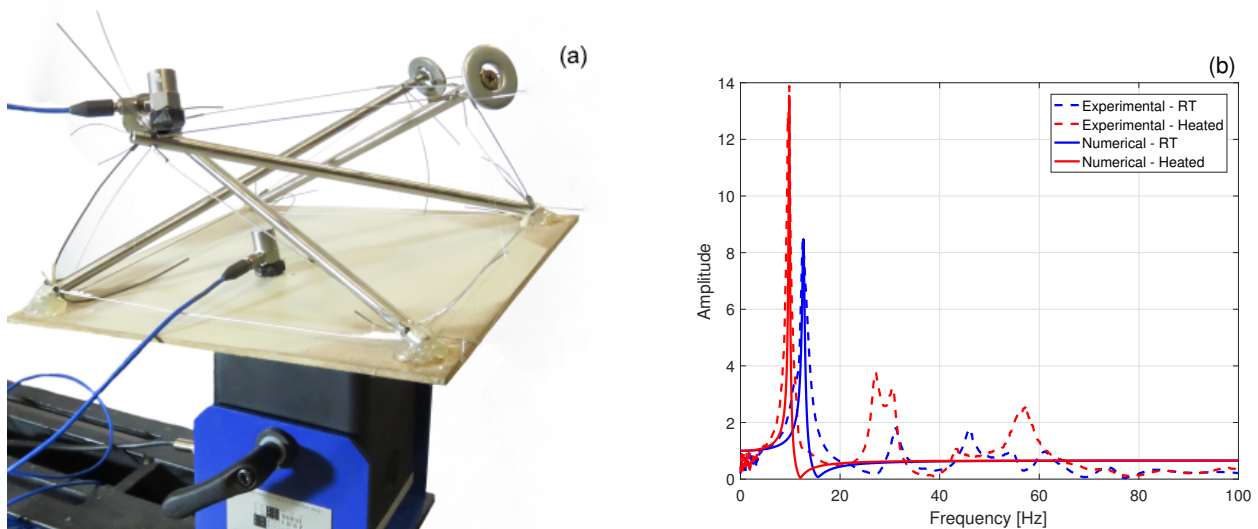
## EXPERIMENTAL TESTS

The objective of the experimental tests was to identify the main resonance of the system so the model could be fitted according to the experimental results. In the experimental process two accelerometers and one shaker were used to obtain the dynamic response of the structure. The tensegrity system was connected to the shaker, with one accelerometer in the base of the structure, and the other one coupled in a upper node, like shown by Fig. 2(a).

With the collected data, it was possible to verify frequency response function, with that analysis the numerical model tension adjustment was feasible to get the same results, as shown by Fig. 2. At first, the analysis was conducted using the SMA in the room temperature (RT), further the temperature of the wire was elevated until occurs the transformation from martensite to austenite. In Fig. 2(b) it can be seen a peak shift of the first resonance frequency, and even though the young's modulus, of the heated SMA is higher, the resonance frequency is lower, as a result of the way the model is constructed, and the tendency of the wire to become straight, causing an relaxation on the other cables, lowering the total stiffness. The cables tensions are determined by the length of each cable. The upper cables and the side cables has the same tension, changing with the SMA transformation. The resonance frequency for the RT SMA, is  $\omega_c = 12.5$  [Hz], and with the heated SMA,  $\omega_h = 9.8$  [Hz].

## CONCLUSION

With the analysis in this work, it was possible to inquiry that, the structure can easily be adjusted by changing the tension on its cables. Although the stiffness of the SMA is increasing, as a result of the temperature change, the natural frequency is lowered, due to the construction and the manner that the tensegrity was constructed, leading to a mitigation on the tension of the others cables caused by the pre configuration of the SMA to a straight condition when heated.



**Figure 2 – Experimental tests, (a) Experimental setup and (b) Comparison Experimental and Numerical, at RT and heated SMA**

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